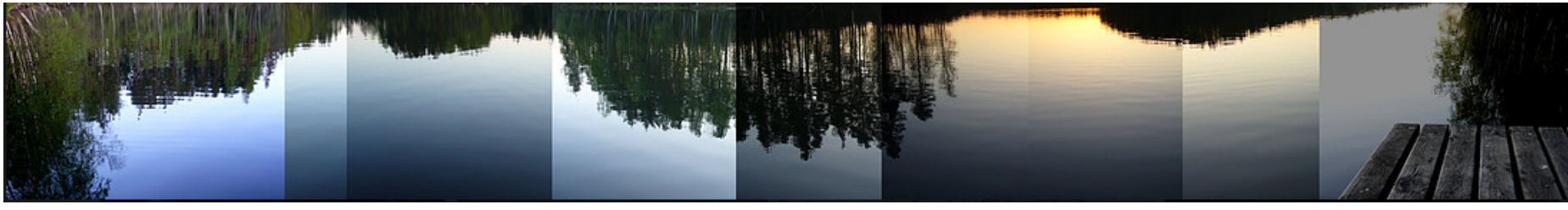
Language Emerging from Meaning Emerging from Language: A Walk in the Logical Woods

Andrés Villaveces
Universidad Nacional de Colombia - Bogotá

Crossing Worlds: Mathematical logic, philosophy, art
An interdisciplinary meeting in honor of Juliette Kennedy.
Helsinki - June 2016



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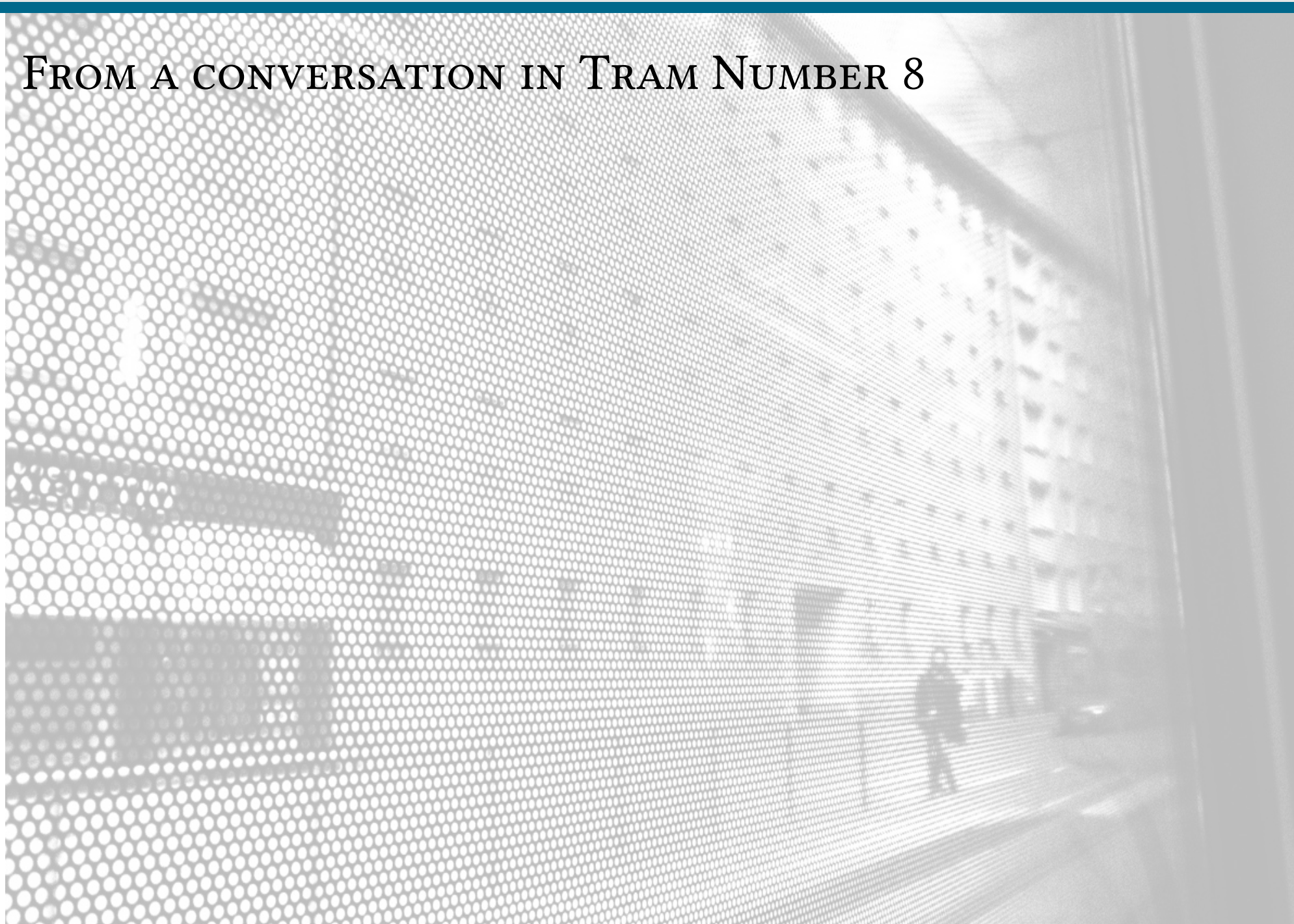
From a conversation in Tram Number 8



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- **JK:** ... Yes, but where exactly do those classes of structures generate their own internal logic? What is their entanglement with logic - those classes are given to us semantically, yet a logic arises. Where from? How exactly? And if the logic was implicit, what frames it? Onto which background is it projected?

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- **AV:** But Juliette, why the deeper question? What drives you to ponder these issues, as if walking far away from a common and secure area? Why your keen insistence on the question?

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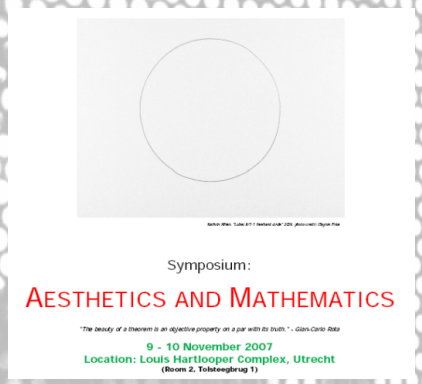
- **AV:** But Juliette, why the deeper question? What drives you to ponder these issues, as if walking far away from a common and secure area? Why your keen insistence on the question?
- **JK:** Well, I want to know **What is Our Standing Point,** as Logicians, as Mathematicians. Go beyond the original ontological discussions that started in 1947 and...

Utrecht, New York, Bogotá, Helsinki, ...



Finding a path / Kilpisjärvi to the Arctic Ocean

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Aesthetics - Logic - Formalism Freeness?

Juliette discusses with students in Bogotá about

- Formalism Freeness?
- Syntax / Semantics?
- Large Cardinals and Definability?
- Political issues?
- Forcing and Invariance?
- Abstraction and Mathematical Drawing?

Syntax / Semantics

Syntax / Semantics Aesthetics?

Syntax / Semantics Aesthetics? Surprising pendularities!

Our two examples - the two directions

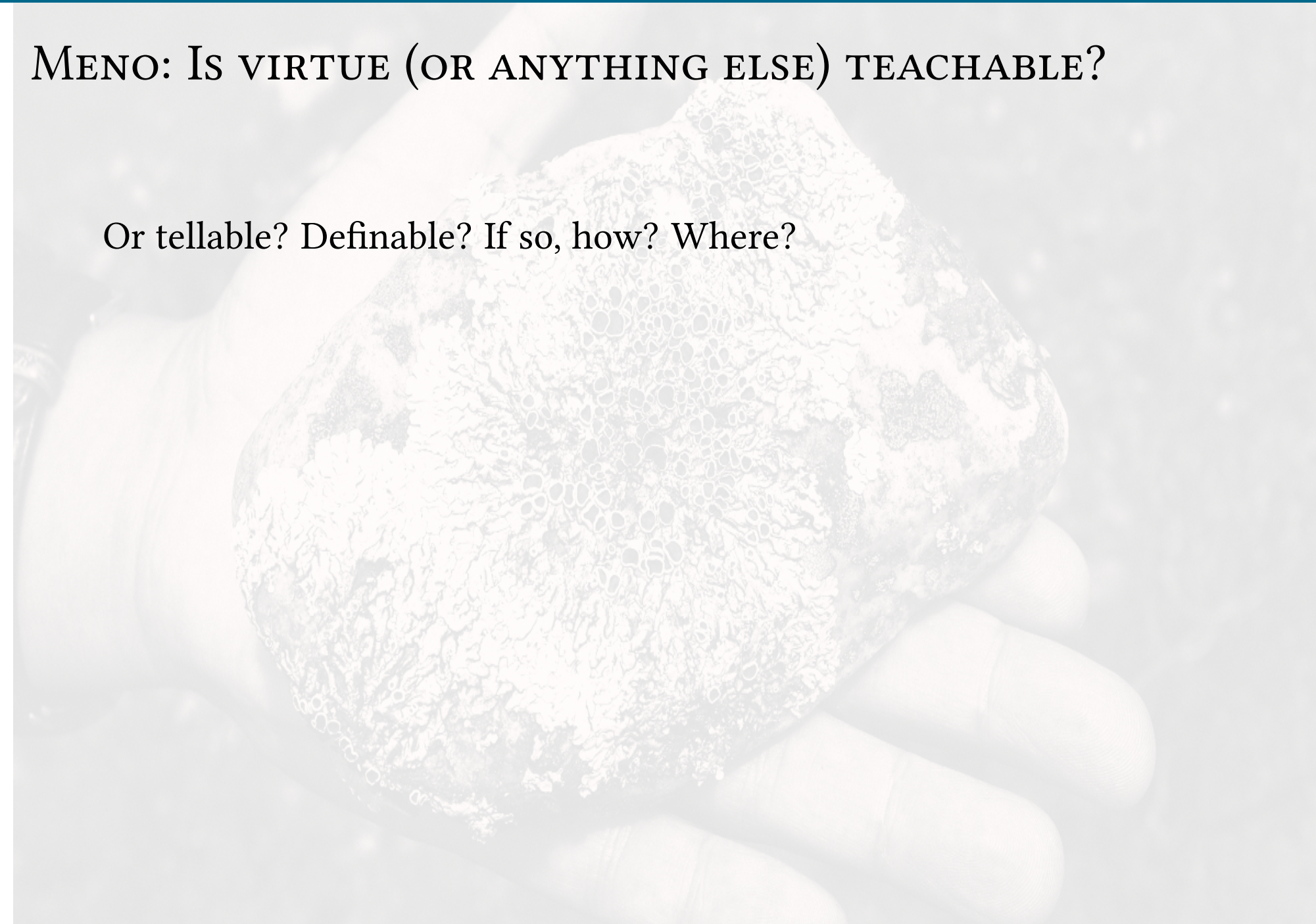
- Inner Models from Extended Logics (Kennedy, Magidor, Väänänen) - extracting (robust) meaning from language?
- Abstract Elementary Classes and the Presentation Theorem - extracting language from (robust) meaning?

Aesthetics and Logic? Grasping!

Jan Zwicky in her Plato as Artist follows the dialogue Meno with an ear to the interplay between Aesthetical, Morality and Phenomenology. Her attention to the problem of grasping as alluded to by Plato/Socrates in that dialogue, in response to the question Is virtue teachable? points toward the bridges we are discussing today, between aesthetics and logic.

Meno: Is virtue (or anything else) teachable?

Or tellable? Definable? If so, how? Where?



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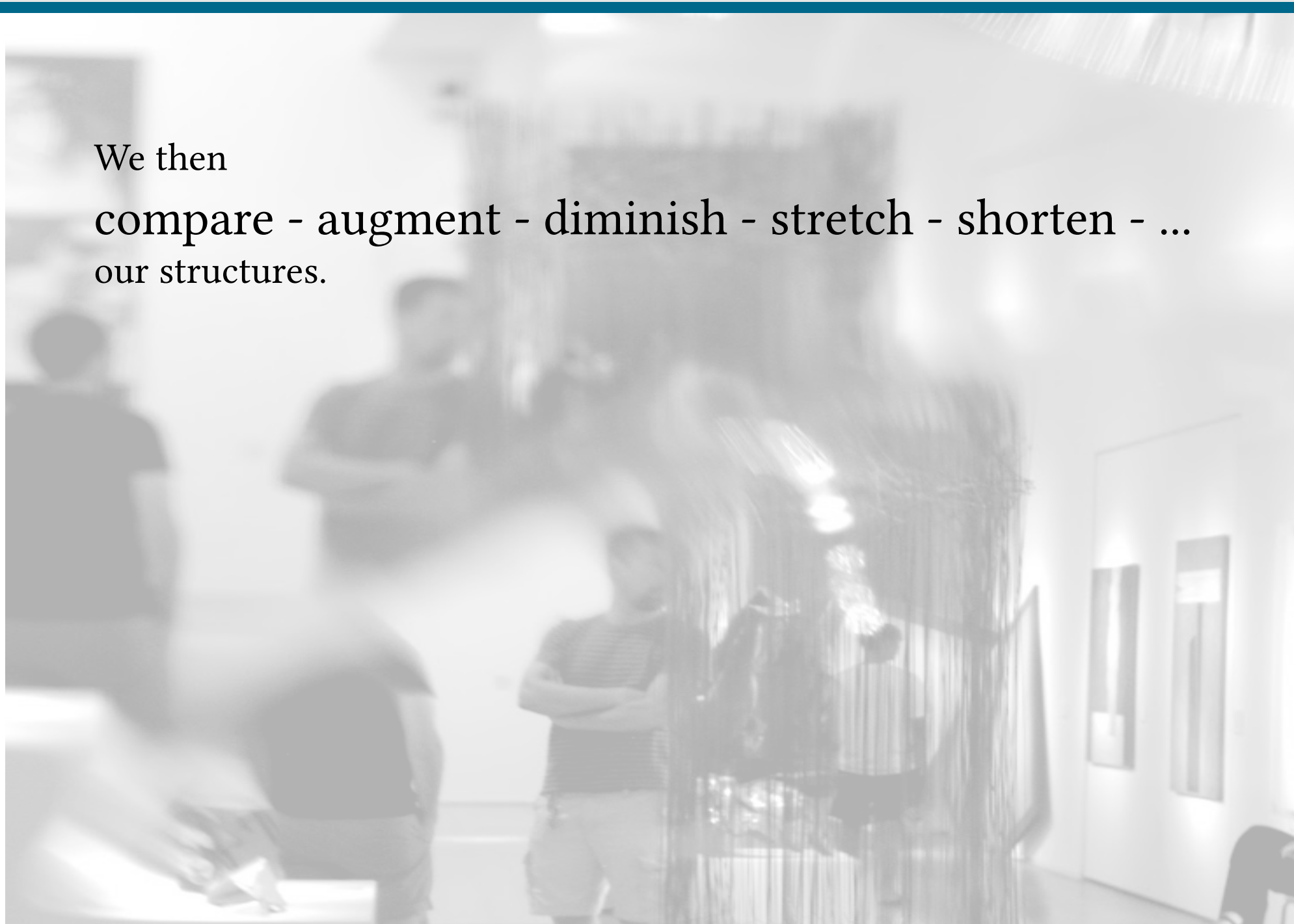
Grasping

The first pendular move: Meaning, from Language

Logical “eye” vs Geometrical “acting”

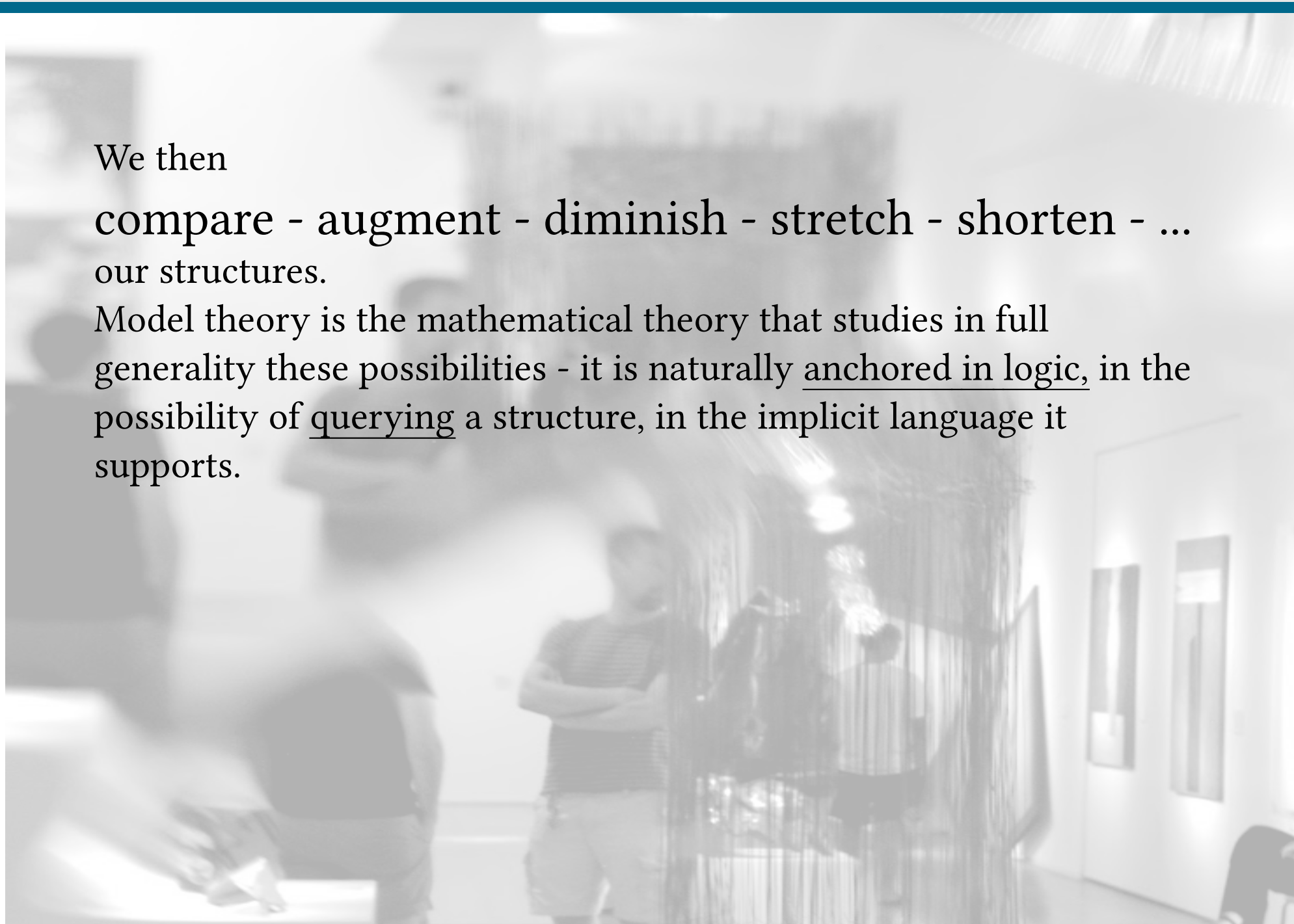
We may “ask questions” to structures: can you see the tower behind you in a Renaissance painting? is it ok to resolve your sonata in C major if you started in D minor? Can we travel back to yesterday? If so, how?



We then

compare - augment - diminish - stretch - shorten - ...

our structures.



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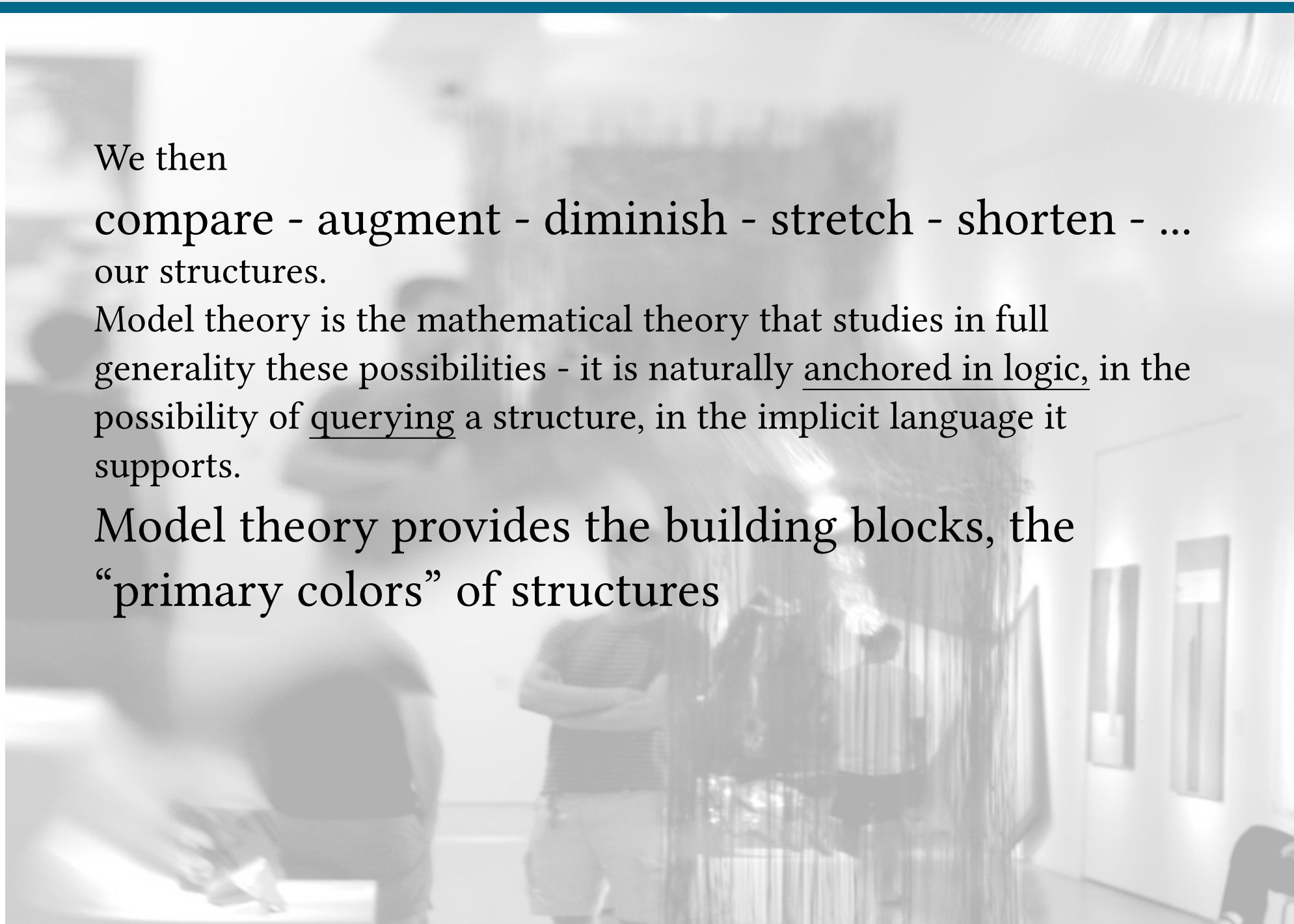
Model theory is the mathematical theory that studies in full generality these possibilities - it is naturally anchored in logic, in the possibility of querying a structure, in the implicit language it supports.



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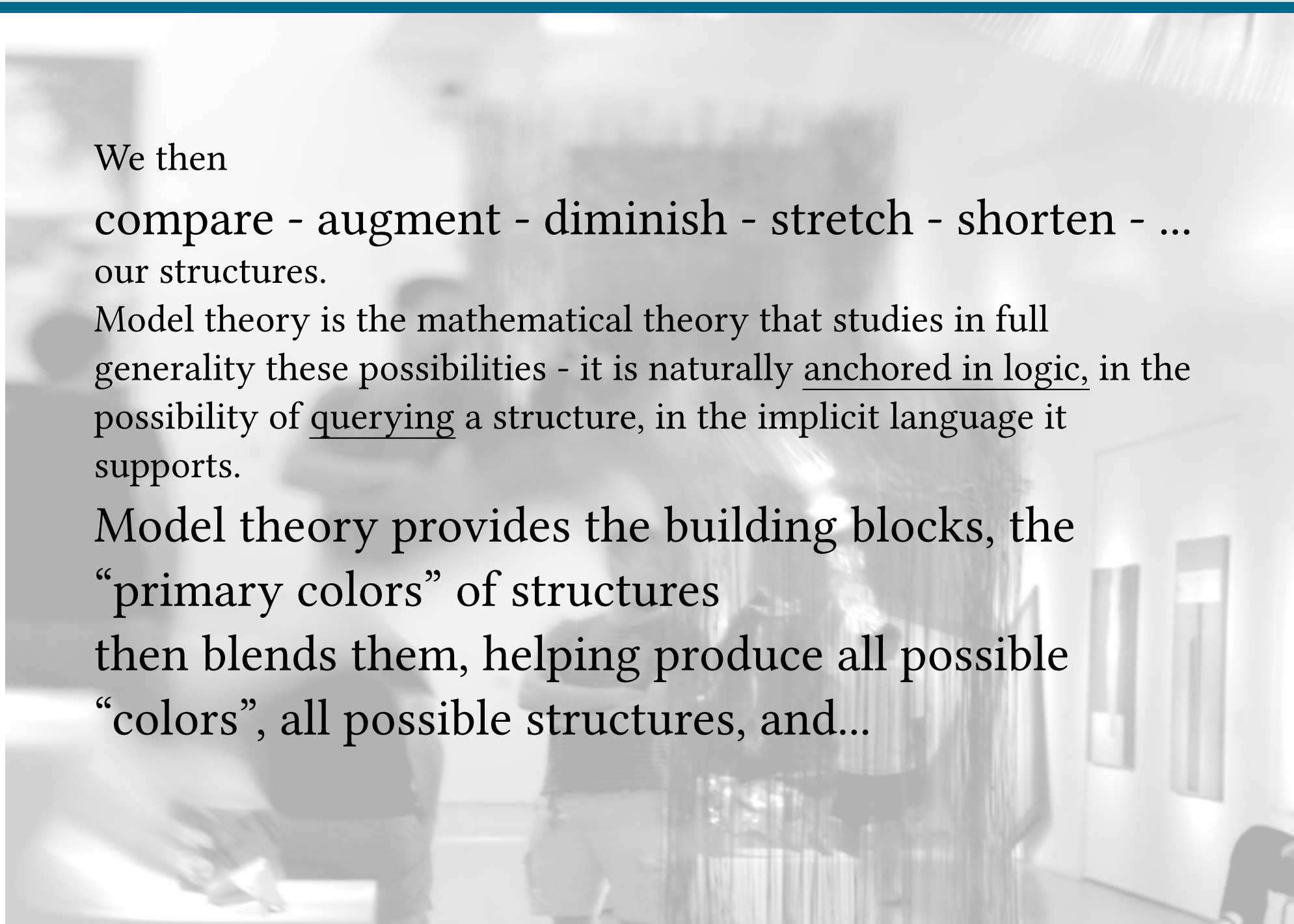

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Model theory provides the building blocks, the "primary colors" of structures

then blends them, helping produce all possible "colors", all possible structures, and...

Yet, surprisingly

in recent years - after Model Theory sharpened its own logical seeing to the point of providing a classification of all possible (first order) theories and asymptotic dividing lines (the Main Gap), it embarked itself into a second sailing, towards the side of “action”, towards geometry, apparently away from logic!

But really?

"We have in mind mainly those interested in algebraically-minded model theory, i.e. in generic models, the class of existentially closed models and universal-homogeneous models rather than elementary classes ..."

Saharon Shelah, 1975.

Inner Models for Different Logics



Emergence of freedom from formalism can be seen in the work of Kennedy, Magidor and Väänänen...

Inner Models extracted from Many Logics

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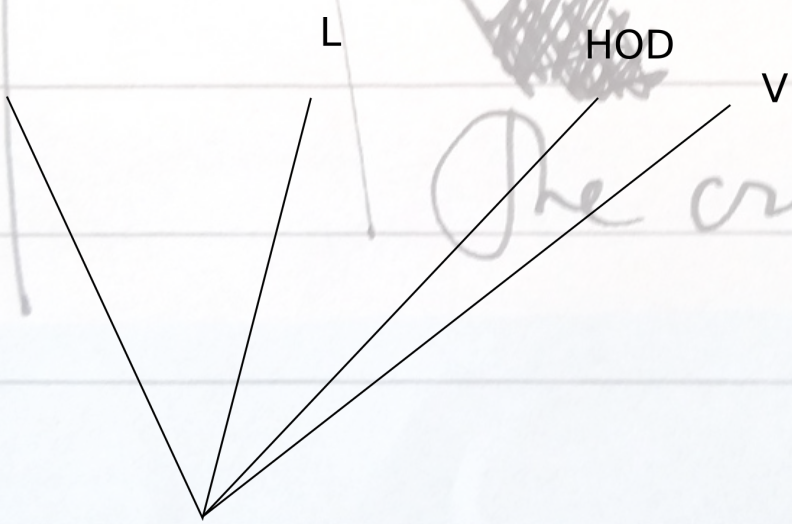

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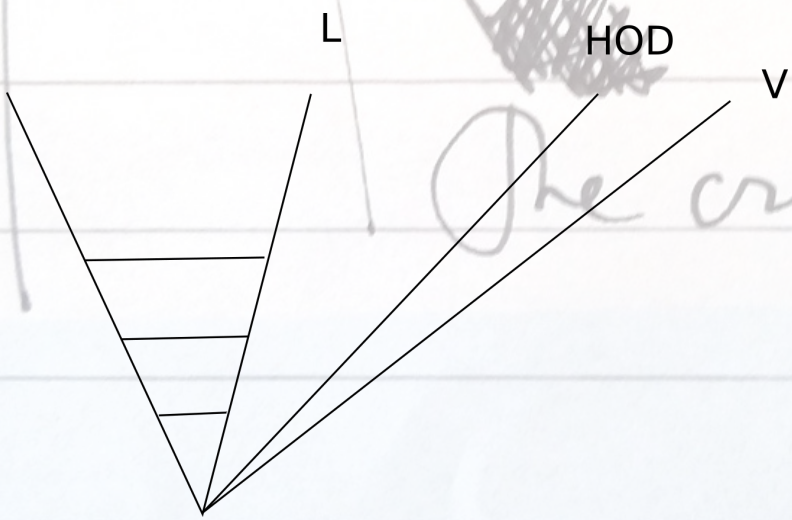

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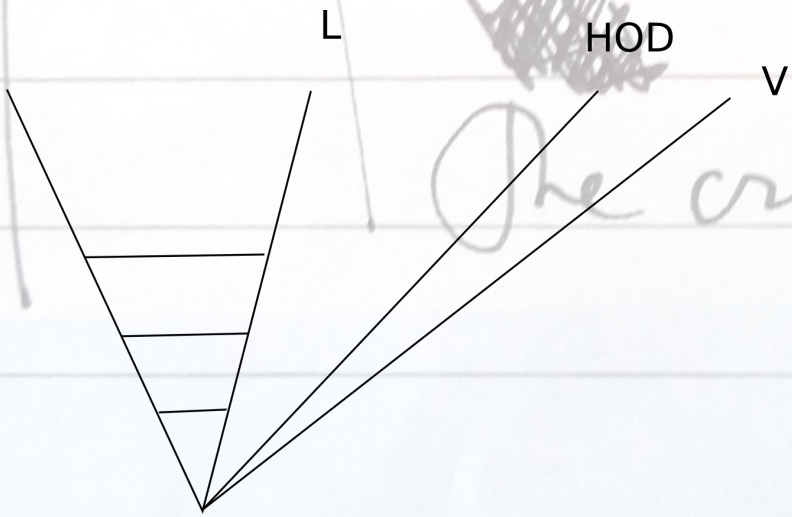

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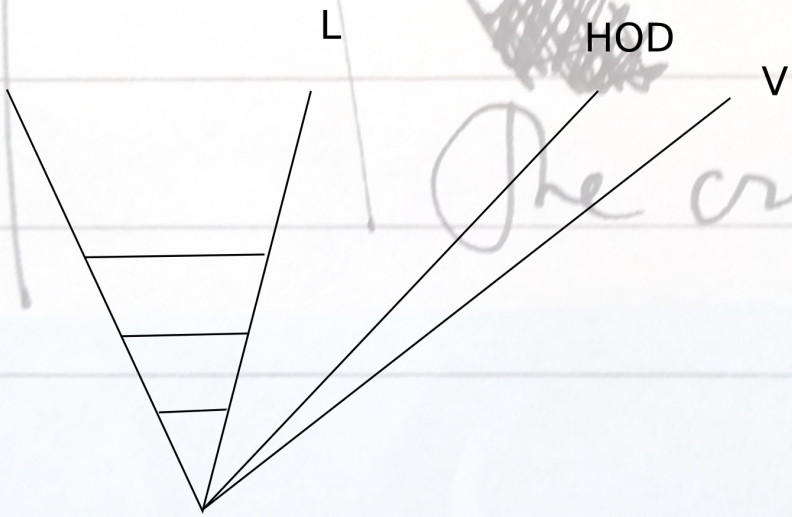

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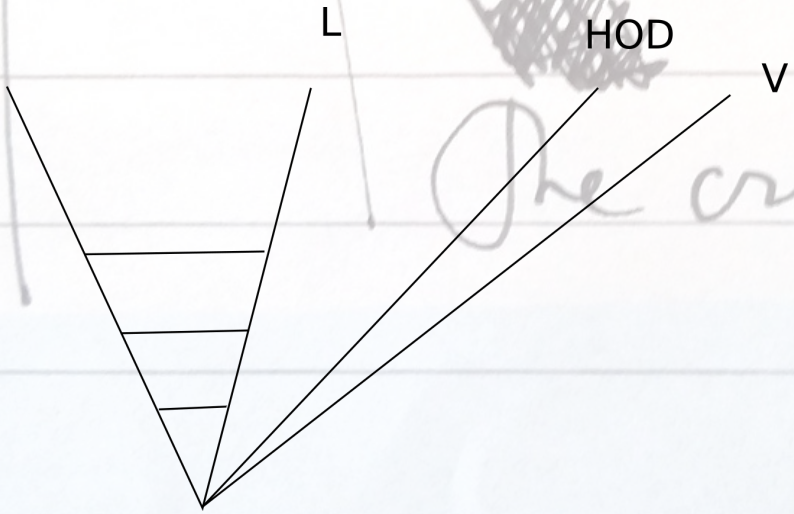

The model L has

- Very tight "internal structure"
- CH: $2^{\aleph_0} = \aleph_1$,
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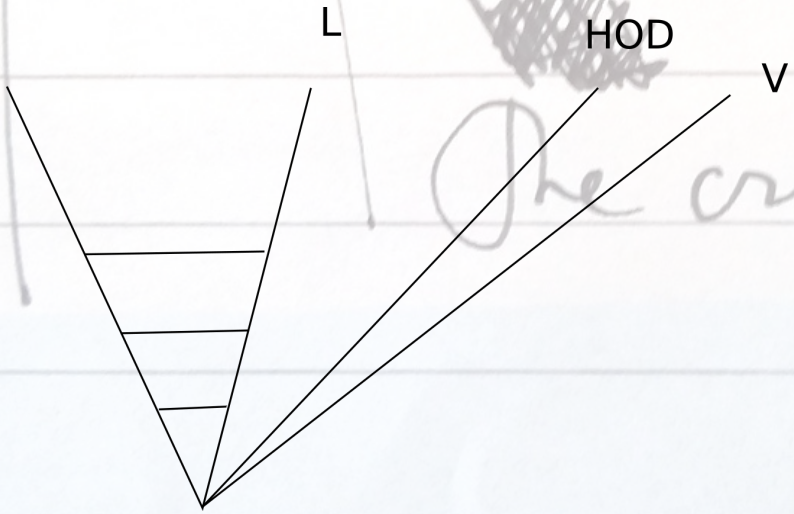

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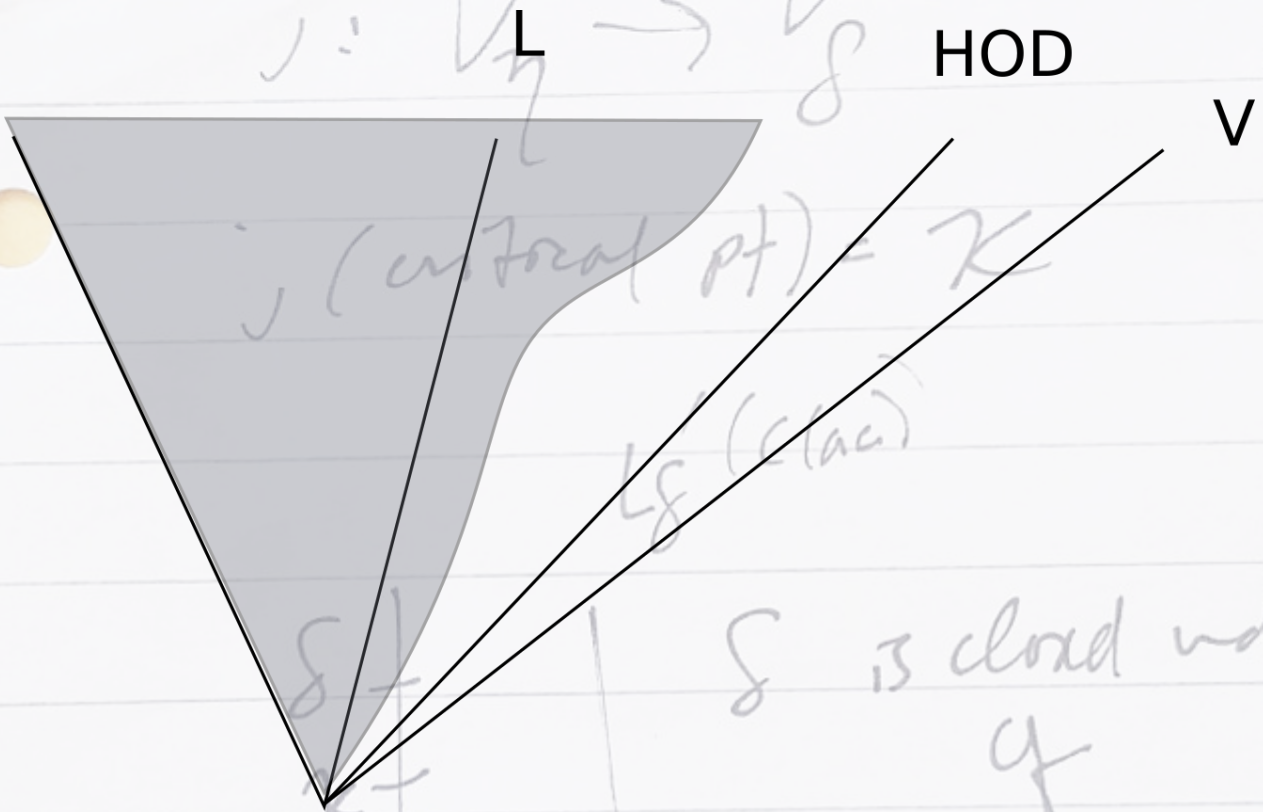
However, L is a rather "narrow" and extreme universe of set theory, lacking many constructions that it would be desirable to have. Therefore L has extreme structure but not enough objects.

Inner Models extracted from Many Logics

- L: Iterating Def, definability in first order
- HOD: Iterating Def^2, definability in second order

Inner Models extracted from Many Logics

- L: Iterating Def, definability in first order
- HOD: Iterating Def^2, definability in second order
- (Kennedy, Magidor, Väänänen): Iterating definability in different logics, as a way to "test" the logics and obtain new inner models.



Robustness - Semantics taking over?

L	C^*	HOD
First Order Logic	Cofinality Quantifier	Second Order Logic

Surprising independence from logic, a very decisive "geometrization away" from logic. But the basis was logic. ?!?!?

(Footnote for mathematicians: under a proper class of measurable Woodin cardinals, KMV prove that regular cardinals are measurable in $C(\mathtt{aa})$, the "version of L" obtained by using definability in stationary logic $L(\mathtt{aa})$ and the theory of $C(\mathtt{aa})$ is invariant under set forcing)

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Robust meaning seems to supercede, to bypass language. Blunt logical seeing? What happened to our sharp tools, formulas and theories? Is logic an illusion of geometry?

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The second sailing of model theory? From λόγος to ἁρμόττον

Back to aesthetics: a concept better suited (Jan Patočka) for aesthetics than "beauty" (τὸ καλὸς),

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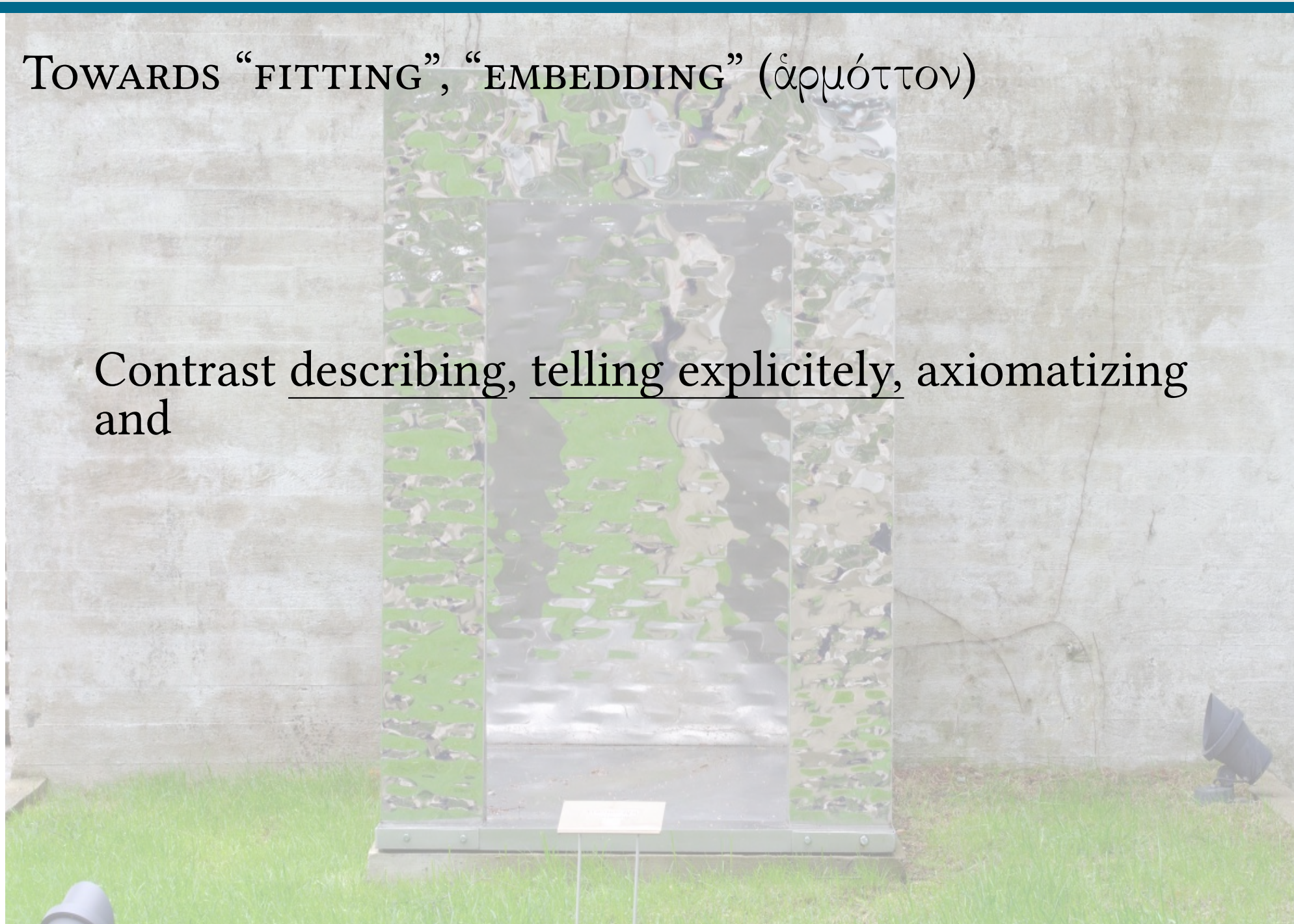
Root: the same as that of the better known ἁρμονία, our harmony, but meaning beauty as "good fitting," "encasing," "embedding." This category seems on the surface to be radically different from that of τὸ λόγος, the phrase, the formula, the description we normally associate with logic.

Carving structures and language - Abstract Elementary Classes

Towards "fitting", "embedding" (ἁρμόττον)



Contrast describing, telling explicitely, axiomatizing and

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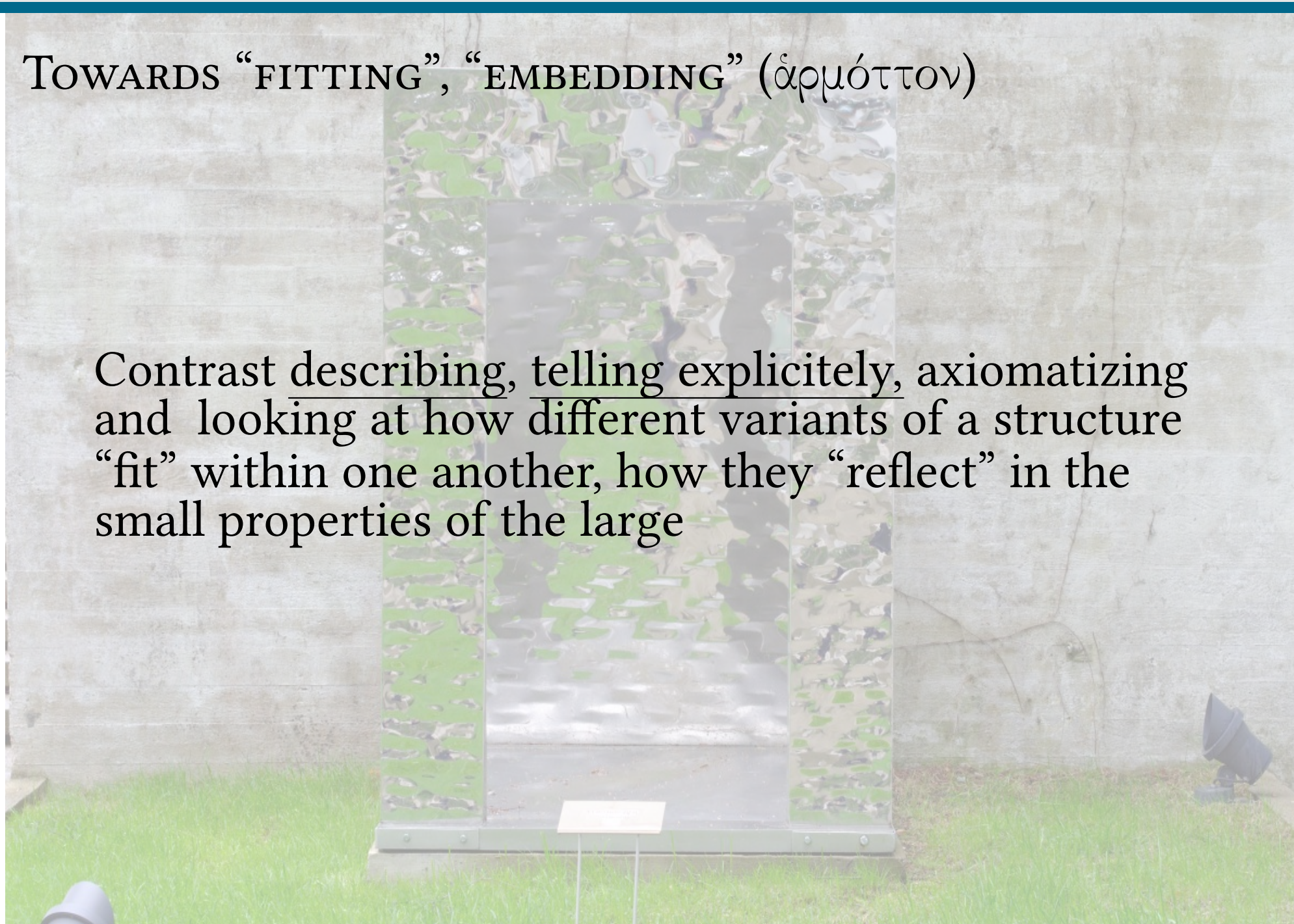


Contrast describing, telling explicitely, axiomatizing and looking at how different variants of a structure "fit" within one another, how they "reflect" in the small properties of the large

Change of Focus: from formulas to embeddings

$\varphi, T, \ldots$	
formulas, theories	

Change of Focus: from formulas to embeddings

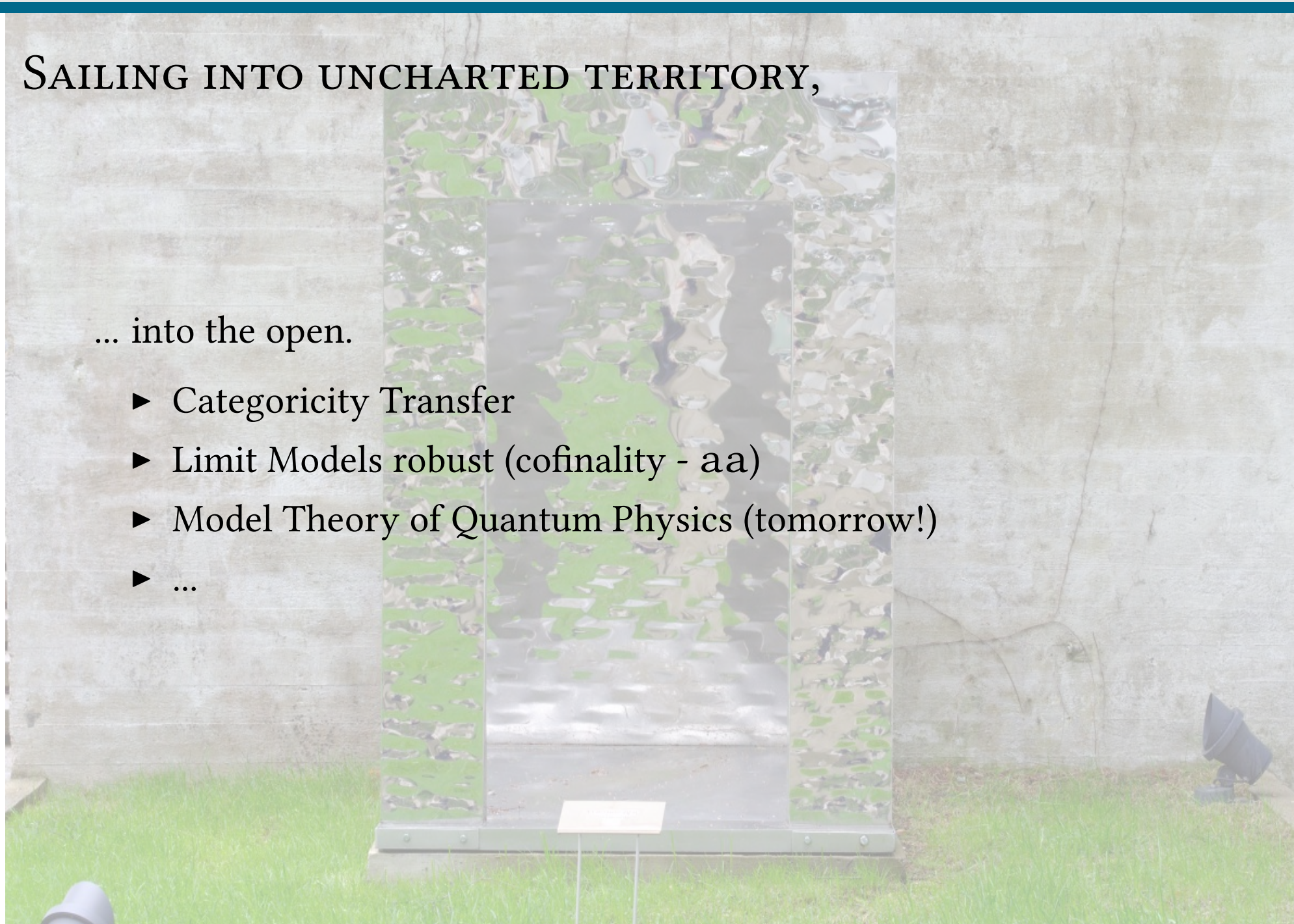
$\varphi, T, \ldots$	$\prec_K$
formulas, theories	embeddings, encasings …

A World of Pure Phenomena...

... without precise descriptions, apparently, but with a strong notion of how pieces are fit within one another—ἁρμόττον

The name for that in contemporary model theory is "strong extension" $M \prec_{\mathcal{K}} N$.

Roughly: all small configurations/problems from M that have a solution in N also have another solution in M.



Sailing into uncharted territory,

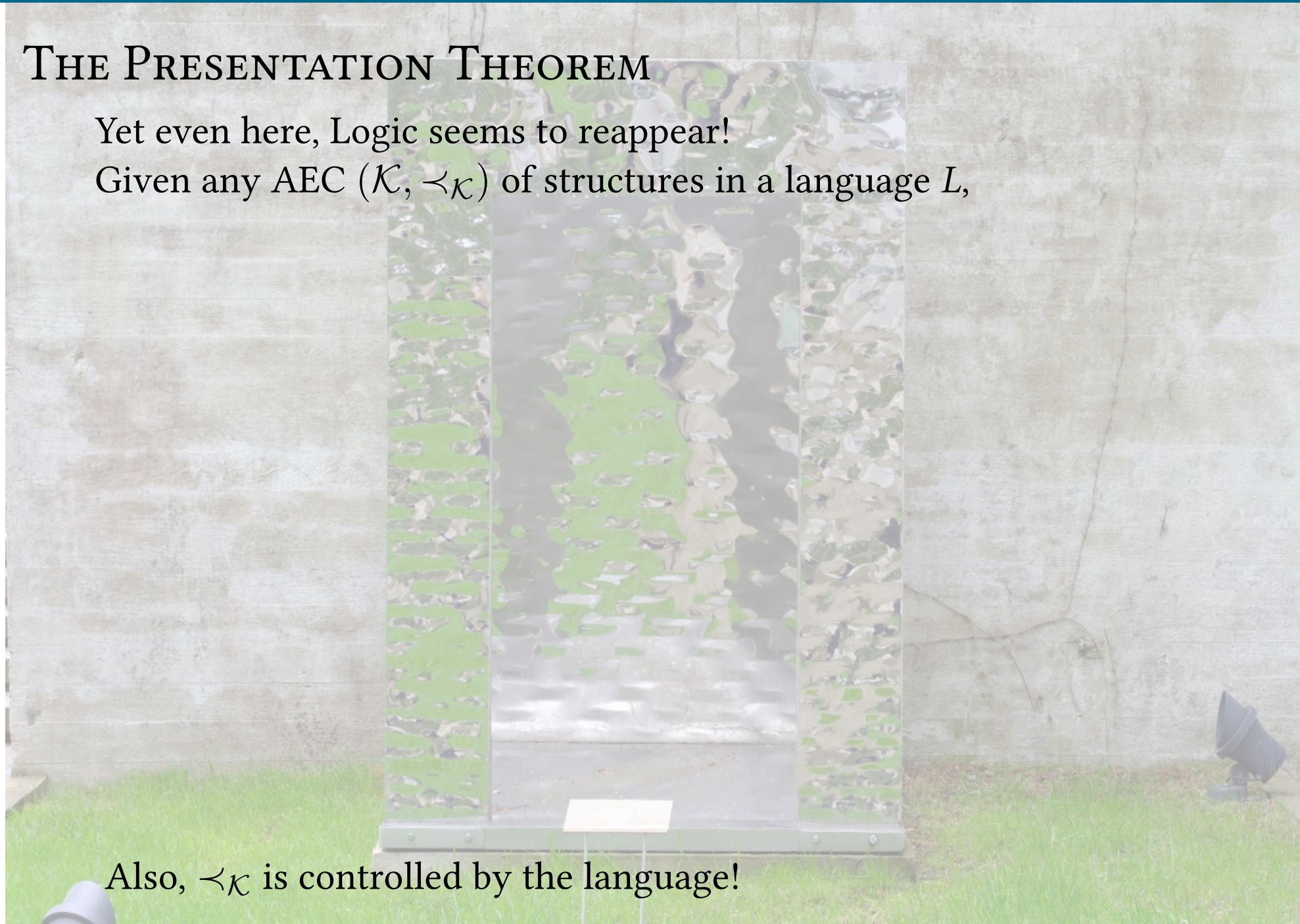
... into the open.

- Categoricity Transfer
- Limit Models robust (cofinality - aa)
- Model Theory of Quantum Physics (tomorrow!)
- ...

The Presentation Theorem

Yet even here, Logic seems to reappear!

Given any AEC $(\mathcal{K}, \prec_{\mathcal{K}})$ of structures in a language L,



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The Presentation Theorem

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Given any AEC $(\mathcal{K}, \prec_{\mathcal{K}})$ of structures in a language L,

there exists a bigger language $L' \supset L$ in which you can write an

infinitary formula ψ that "holds" all the information on the class $\mathcal{K}$ -

in our local dialect we say that $\mathcal{K}$ is a PC-class for omitting types in

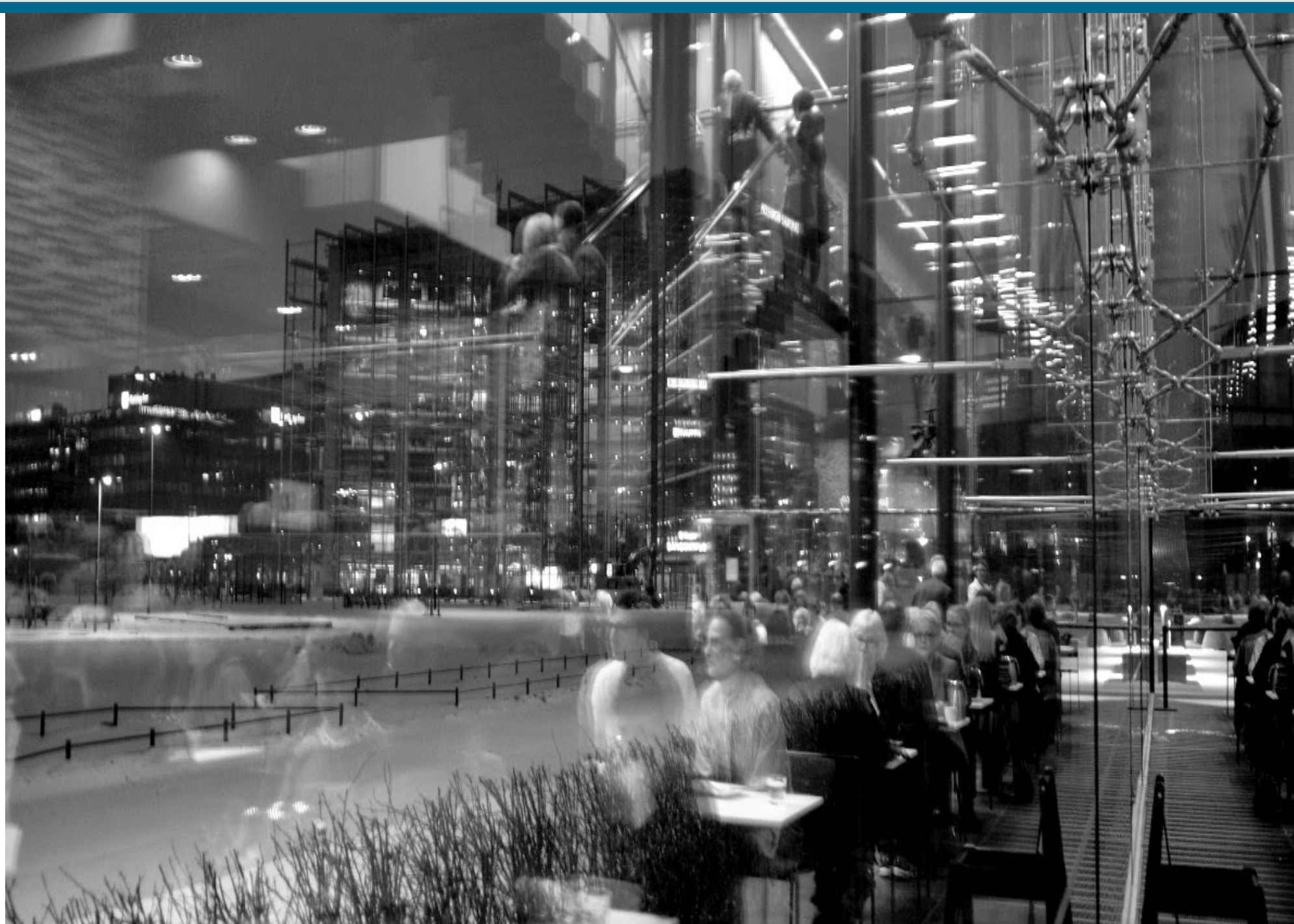
an expanded language,

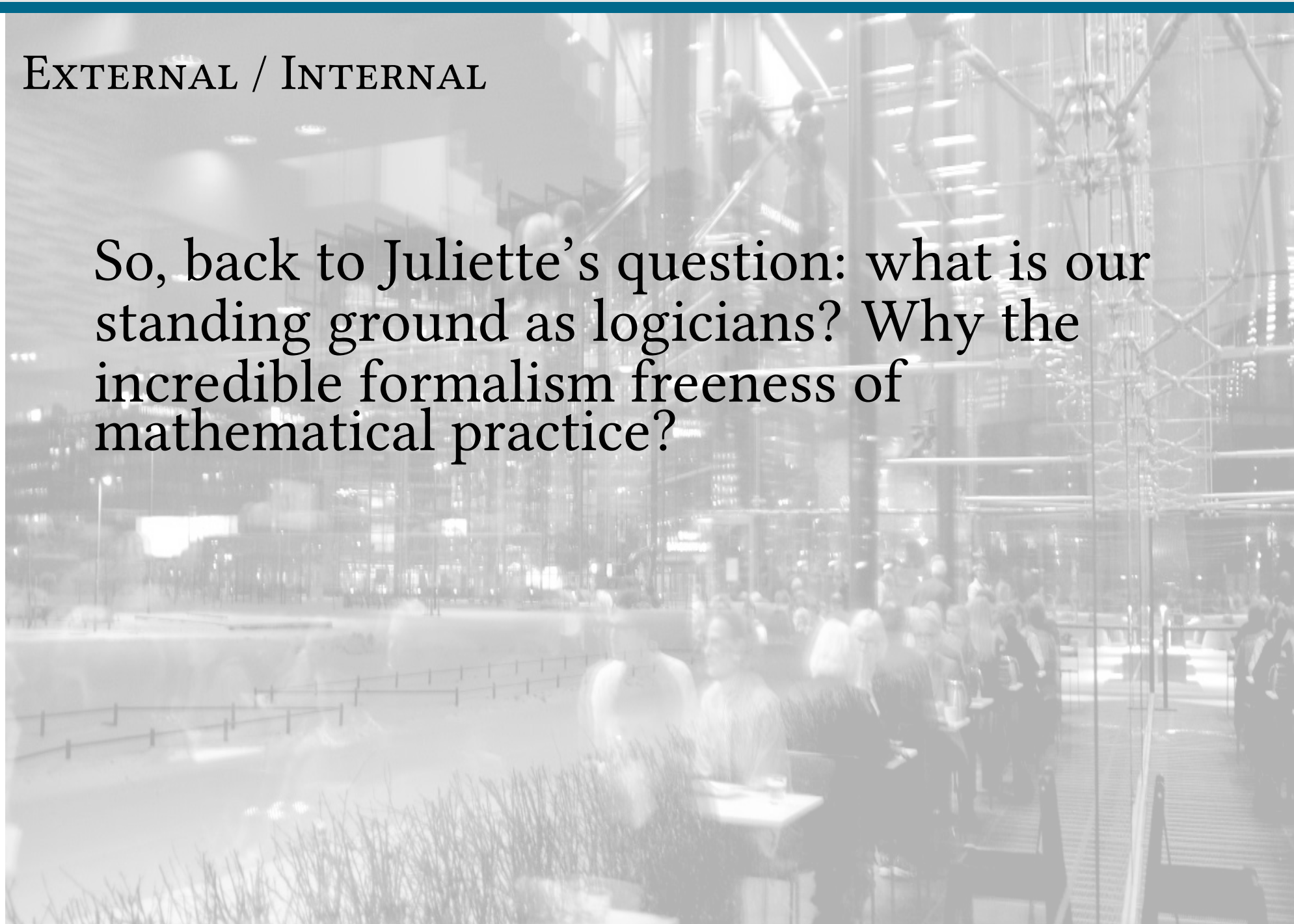
$$\mathcal{K} = PC(L, T', \Gamma')$$

Also, $\prec_{\mathcal{K}}$ is controlled by the language!

Pure phenomena? No logos? Well...

The second sailing is thus firmly anchored in territory we may chart - albeit indirectly. This new Model Theory does not “reduce” to a class controled by the logic. In practice, the theorem provides “steam” (in our lingo, Ehrenfeucht-Mostowski models, for example) to carry long-winded constructions in the absence of the tools (compactness) present in first order.





External / Internal

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Thank you all!