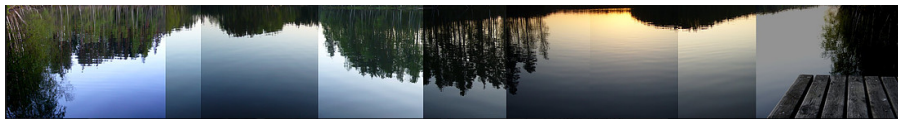


Language Emerging from Meaning Emerging from Language: A Walk in the Logical Woods

Andrés Villaveces

Universidad Nacional de Colombia - Bogotá

Crossing Worlds: Mathematical logic, philosophy, art
An interdisciplinary meeting in honor of Juliette Kennedy.
Helsinki - June 2016



CONTENTS

Syntax vs Semantics in Aesthetics?

Aesthetics, Syntax, Semantics

A word on Aesthetics

Meaning from language

Logical “seeing” vs geometrical “acting”

Inner Models for Different Logics: extracting meaning from language /

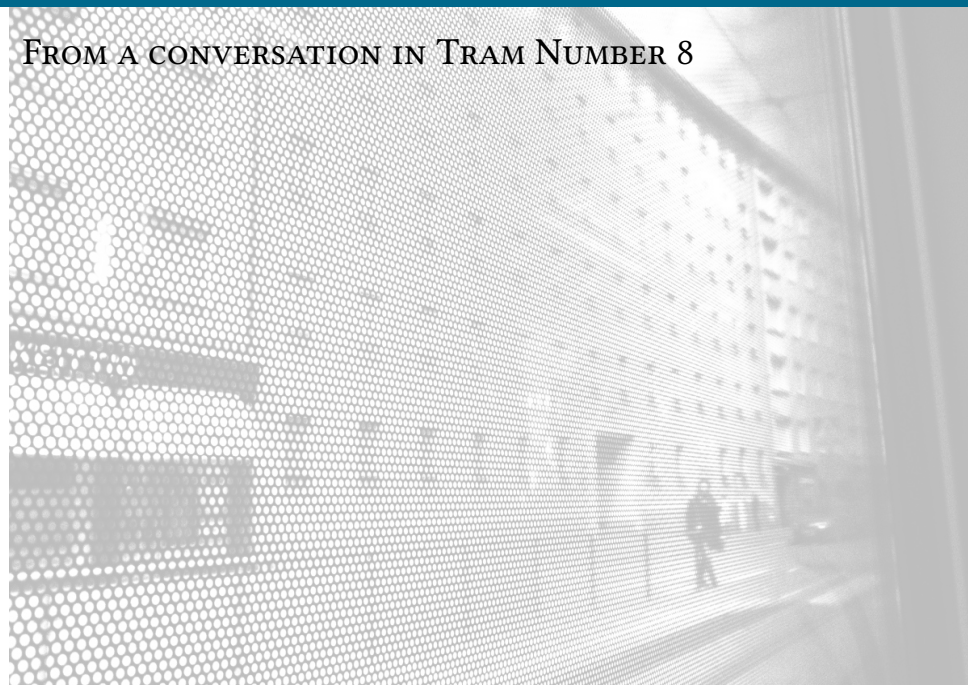
Language from meaning

The contrast between *lógos* and *harmóton*

Carving structures and language: Abstract “Elementary” Classes

Finale: So, where do we stand as logicians?

FROM A CONVERSATION IN TRAM NUMBER 8



FROM A CONVERSATION IN TRAM NUMBER 8

- **JK:** ... Yes, but where exactly do those classes of structures generate their own internal logic? What is their entanglement with logic - those classes are given to us semantically, yet a logic arises. Where from? How exactly? And if the logic was implicit, what frames it? Onto which background is it projected?

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- ▶ **AV:** But Juliette, why the deeper question? What drives you to ponder these issues, as if walking far away from a common and secure area? Why your keen insistence on the question?
- ▶ **JK:** Well, I want to know **What is Our Standing Point**, as Logicians, as Mathematicians. Go beyond the original ontological discussions that started in 1947 and...

Getting There and Falling Short

Please register by May 15th at www.conference.sagepub.com

The interdisciplinary symposium **Getting There and Staying There** looks at some of the difficulties surrounding the idea of complex control. We consider the issue from the perspectives of virtual disciplines (cognitive, mechanical, mathematical, chemical and robotic) and the "topping strategies" approach. How do these two modes of action take advantage of complex control?

Intelligence complexity, linguistic knowledge, the influence of external model, horizons of responsibility, list of external inputs of what is external, representational and computational systems (and control aspects, how do the different disciplines deal with the dynamic of getting there and staying there)?

Photo by Maria Elena Cortes. "Marine Head" from the series *Conversations Between Two and Nothingness*, 2003.



¹²The image of a theorem is an abstract property on a set with its truth. (See Clark 1996)

9 - 10 November 2007
Location: Louis Hartlooper Complex, Utrecht
(Bosch 2, Tolstegebaan 1)



1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

8, 10 y 11 de noviembre de 2014

Ignacio Arce (Bogotá)	Maria Margarita Manago (Bogotá)
John Salazar (Chicago)	Alejandra Merin (Cali)
Kater (Socorro) (Bogotá)	Bernardo Ortiz (Bogotá)
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Julieta Kennedy (Helsinki)	

+ una exposición de arte

(Tercer piso del Museo de Arquitectura Leopoldo Rether)



FINDING A PATH / KILPISJÄRVI TO THE ARCTIC OCEAN



FINDING A PATH / KILPISJÄRVI TO THE ARCTIC OCEAN



Aesthetics - Logic -
Formalism Freeness?

Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?

Meaning from language

Language from meaning

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JULIETTE DISCUSSES WITH STUDENTS IN BOGOTÁ ABOUT

- ▶ Formalism Freeness?
- ▶ Syntax / Semantics?
- ▶ Large Cardinals and Definability?
- ▶ Political issues?
- ▶ Forcing and Invariance?
- ▶ Abstraction and Mathematical Drawing?

Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?

Meaning from language

Language from meaning

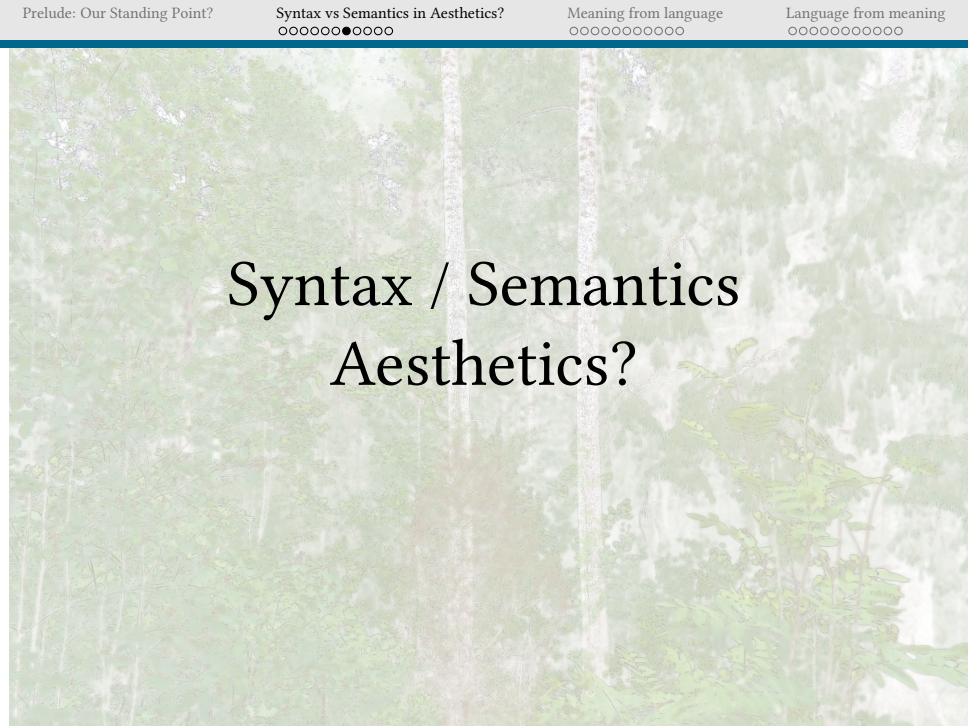
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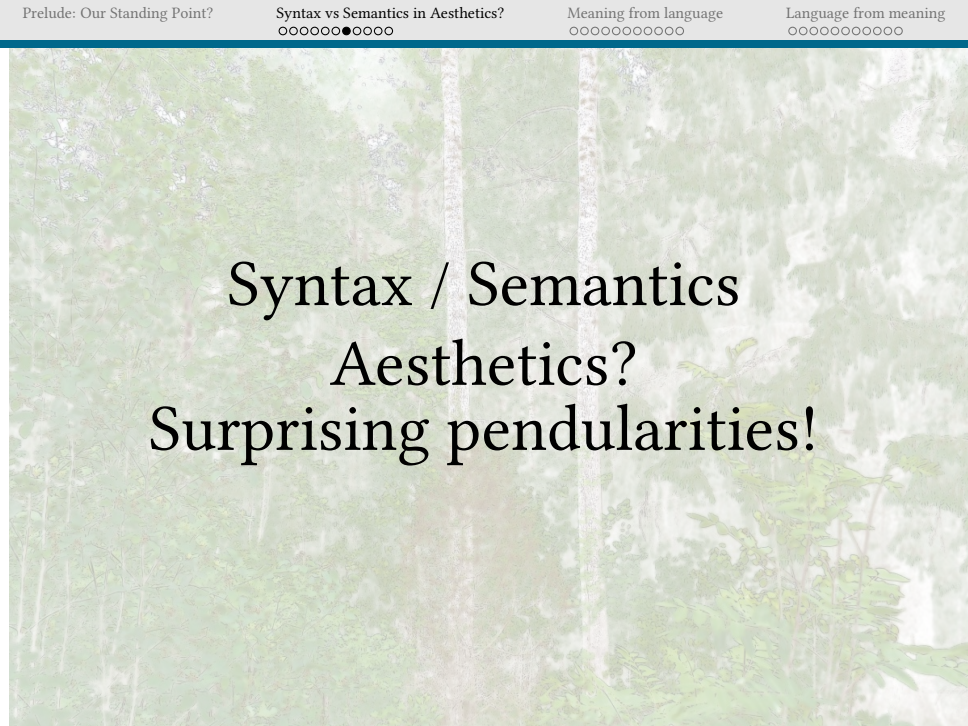
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Syntax / Semantics



Syntax / Semantics Aesthetics?



Syntax / Semantics Aesthetics? Surprising pendularities!

OUR TWO EXAMPLES - THE TWO DIRECTIONS

- ▶ Inner Models from Extended Logics (Kennedy, Magidor, Väänänen) - extracting (robust) meaning from language?
- ▶ Abstract Elementary Classes and the Presentation Theorem - extracting language from (robust) meaning?

Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?

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Language from meaning

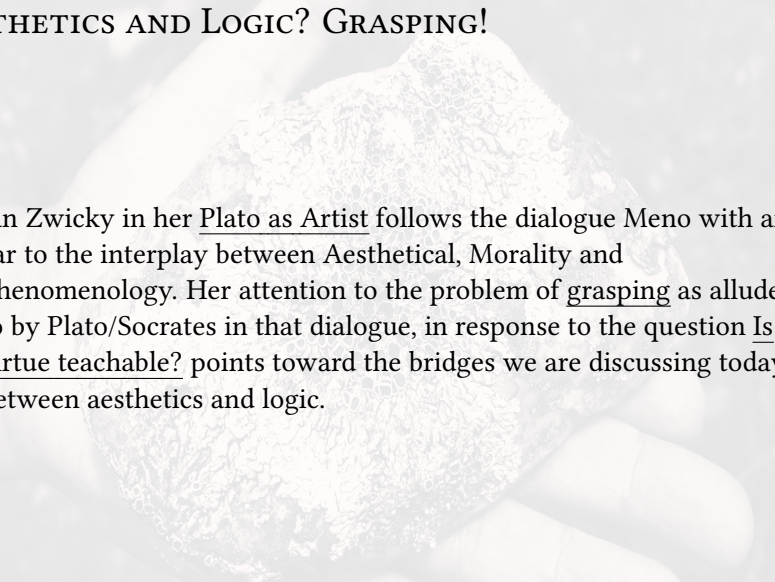
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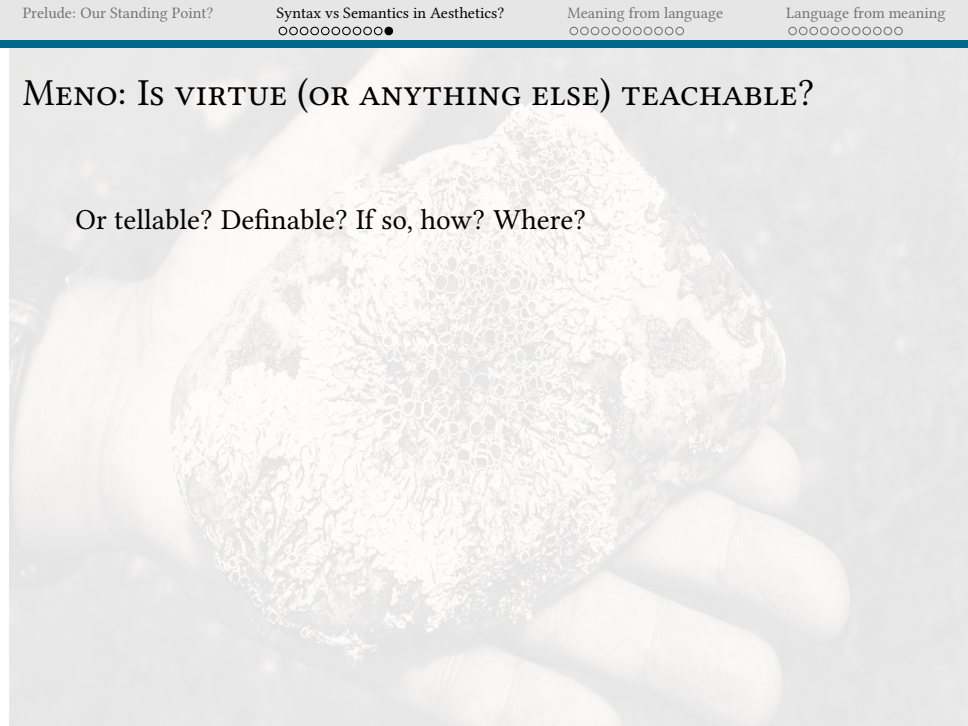
AESTHETICS AND LOGIC? GRASPING!

A background image showing a hand holding a rough, textured rock, possibly a piece of coral or a fossil, with a complex, porous surface. The hand is positioned as if presenting the object.

Jan Zwicky in her Plato as Artist follows the dialogue Meno with an ear to the interplay between Aesthetical, Morality and Phenomenology. Her attention to the problem of grasping as alluded to by Plato/Socrates in that dialogue, in response to the question Is virtue teachable? points toward the bridges we are discussing today, between aesthetics and logic.

MENO: IS VIRTUE (OR ANYTHING ELSE) TEACHABLE?

Or tellable? Definable? If so, how? Where?



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From the OED:

The origin of the word aesthetics is αἰσθητικ-ός / αἰσθητά, things perceptible by the senses, from the stem αἰσθε- 'feel, apprehend by the senses'.

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Grasping

THE FIRST PENDULAR MOVE: MEANING, FROM LANGUAGE

Logical “eye” vs Geometrical
“acting”

Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?

Meaning from language

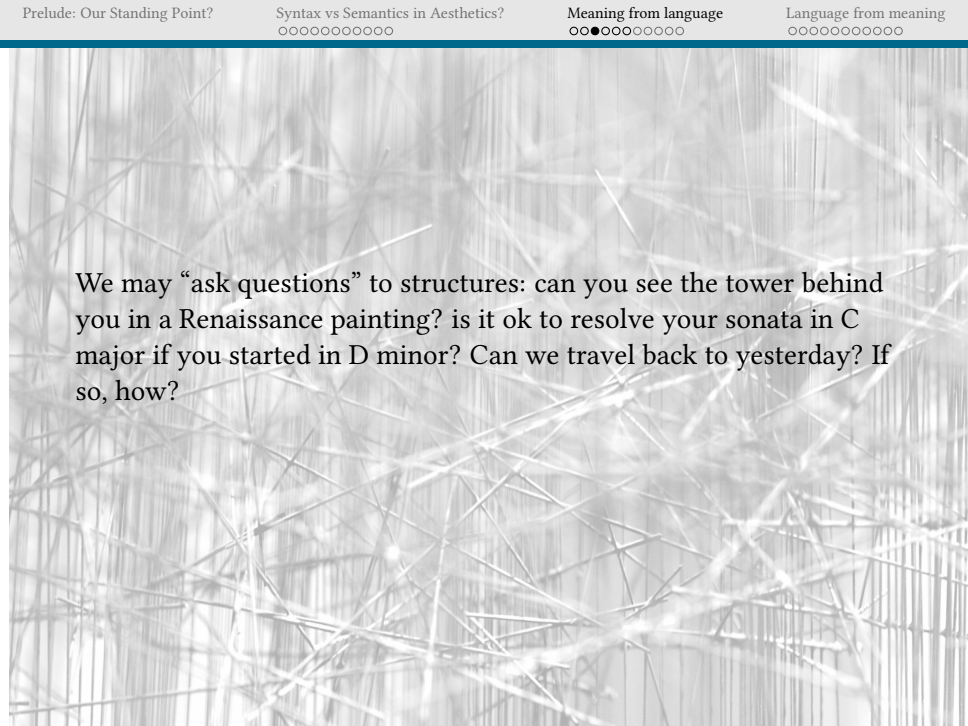
Language from meaning

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We may “ask questions” to structures: can you see the tower behind you in a Renaissance painting? is it ok to resolve your sonata in C major if you started in D minor? Can we travel back to yesterday? If so, how?

We then

compare - augment - diminish - stretch - shorten - ...
our structures.



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Model theory is the mathematical theory that studies in full
generality these possibilities - it is naturally anchored in logic, in the
possibility of querying a structure, in the implicit language it
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Model theory provides the building blocks, the
“primary colors” of structures

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Model theory is the mathematical theory that studies in full generality these possibilities - it is naturally anchored in logic, in the possibility of querying a structure, in the implicit language it supports.

Model theory provides the building blocks, the “primary colors” of structures
then blends them, helping produce all possible “colors”, all possible structures, and...

YET, SURPRISINGLY

in recent years - after Model Theory sharpened its own logical seeing to the point of providing a classification of all possible (first order) theories and asymptotic dividing lines (the Main Gap), it embarked itself into a second sailing, towards the side of “action”, towards geometry, apparently away from logic!

BUT REALLY?

“We have in mind mainly those interested in algebraically-minded model theory, i.e. in generic models, the class of existentially closed models and universal-homogeneous models rather than elementary classes ...”

Saharon Shelah, 1975.

Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?

Meaning from language

Language from meaning

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INNER MODELS FOR DIFFERENT LOGICS



Emergence of freedom from formalism can be seen in the work of Kennedy, Magidor and Väänänen...

INNER MODELS EXTRACTED FROM MANY LOGICS

► $L_0 = \emptyset$

L

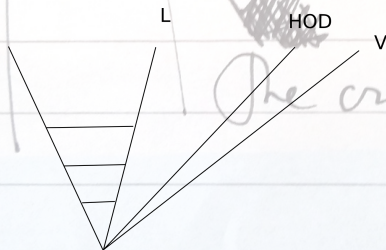
HOD

V

Sometimes you do your best but
find a way to make make better
the maddest thing.

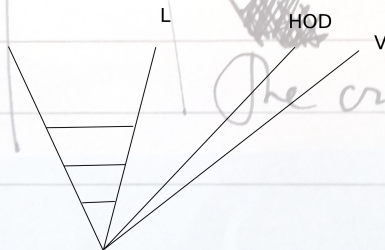
INNER MODELS EXTRACTED FROM MANY LOGICS

- ▶ $L_0 = \emptyset$
- ▶ $L_{\alpha+1} = \text{Def}^1(L_\alpha)$



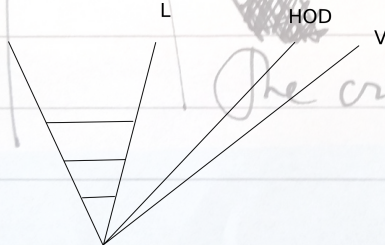
INNER MODELS EXTRACTED FROM MANY LOGICS

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- ▶ $L_\delta = \bigcup_{\alpha < \delta} L_\alpha$



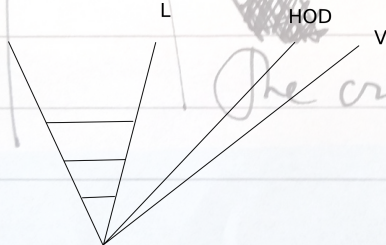
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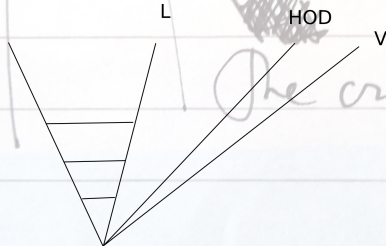
The model L has

- ▶ Very tight “internal structure”
- ▶ CH: $2^{\aleph_0} = \aleph_1$,
- ▶ GCH: $2^{\aleph_\alpha} = \aleph_{\alpha+1}$ for all α
- ▶ And much more

Sometimes you do your best but
 find a way to make make better
 the maddest inquiry.

INNER MODELS EXTRACTED FROM MANY LOGICS

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The model L has

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- ▶ And much more

However, L is a rather “narrow” and extreme universe of set theory, lacking many constructions that it would be desirable to have.

Therefore L has extreme structure but not enough objects.

INNER MODELS EXTRACTED FROM MANY LOGICS

$$\exists^{\forall \eta} \eta < K$$

(Major reformulation of supercompactness)

- L : Iterating *Def*, definability in first order
- HOD : Iterating *Def*², definability in second order

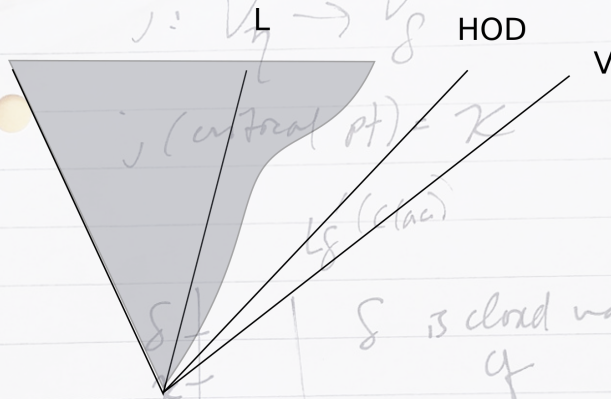
$$j(\text{critical pt}) = K$$

$$L_{\mathcal{S}}'(C(\alpha))$$

$$\delta \perp \kappa \models \quad \delta \text{ is closed under subsets}$$

INNER MODELS EXTRACTED FROM MANY LOGICS

- ▶ L : Iterating Def , definability in first order
- ▶ HOD : Iterating Def^2 , definability in second order
- ▶ (Kennedy, Magidor, Väänänen): Iterating definability in different logics, as a way to “test” the logics and obtain new inner models.



ROBUSTNESS - SEMANTICS TAKING OVER?

L	C^*	HOD
First Order Logic	Cofinality Quantifier	Second Order Logic

Surprising independence from logic, a very decisive “geometrization away” from logic. But the basis was logic. ?!?!?

(Footnote for mathematicians: under a proper class of measurable Woodin cardinals, KMV prove that regular cardinals are measurable in $C(aa)$, the “version of L ” obtained by using definability in stationary logic $L(aa)$ and the theory of $C(aa)$ is invariant under set forcing)

ROBUSTNESS - SEMANTICS TAKING OVER?

L	C^*	HOD
First Order Logic	Cofinality Quantifier	Second Order Logic

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Robust meaning seems to supercede, to bypass language. Blunt logical seeing? What happened to our sharp tools, formulas and theories? Is logic an illusion of geometry?

(Footnote for mathematicians: under a proper class of measurable Woodin cardinals, KMV prove that regular cardinals are measurable in $C(aa)$, the “version of L ” obtained by using definability in stationary logic $L(aa)$ and the theory of $C(aa)$ is invariant under set forcing)

Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?

Meaning from language

Language from meaning

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THE SECOND SAILING OF MODEL THEORY? FROM λόγος TO ἁρμόττον

Back to aesthetics: a concept better suited (Jan Patočka) for
aesthetics than “beauty” (τὸ καλὸς),

τὸ ἁρμόττον.

THE SECOND SAILING OF MODEL THEORY? FROM λόγος TO ἁρμόττον

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Root: the same as that of the better known ἁρμονία, our harmony, but meaning beauty as “good fitting,” “encasing,” “embedding.”

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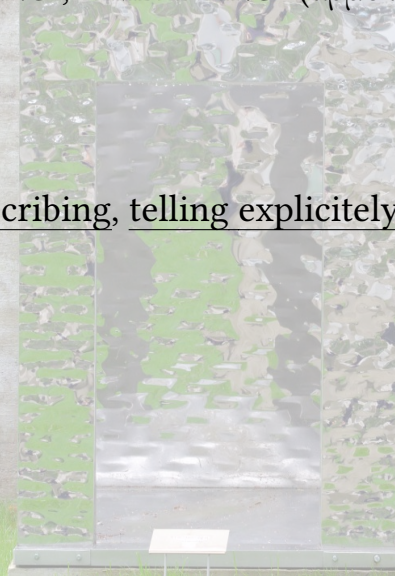
Root: the same as that of the better known ἁρμονία, our harmony, but meaning beauty as “good fitting,” “encasing,” “embedding.” This category seems on the surface to be radically different from that of τὸ λόγος, the phrase, the formula, the description we normally associate with logic.

CARVING STRUCTURES AND LANGUAGE - ABSTRACT ELEMENTARY CLASSES



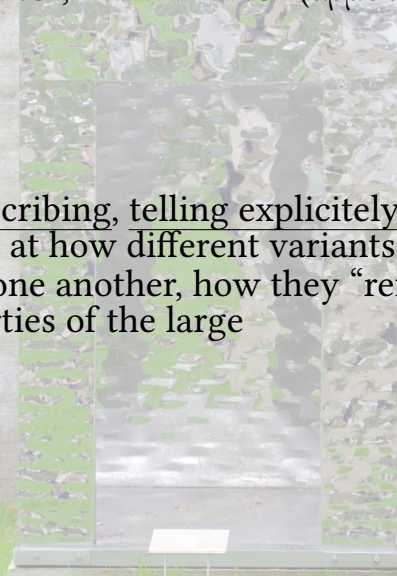
TOWARDS “FITTING”, “EMBEDDING” (ἄρμόττον)

Contrast describing, telling explicitly, axiomatizing
and

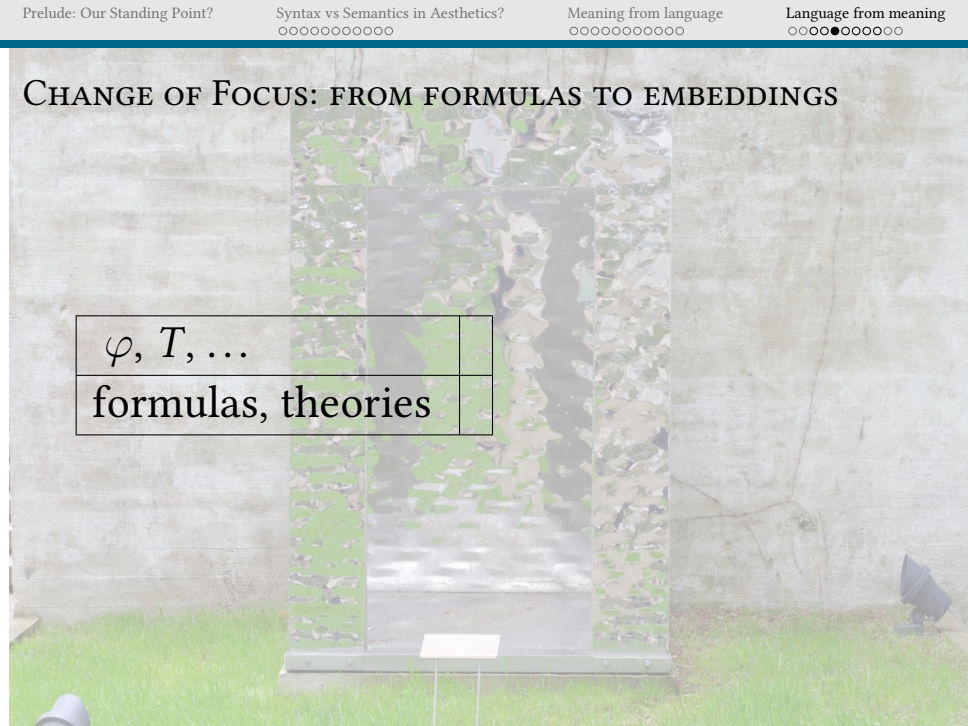


TOWARDS “FITTING”, “EMBEDDING” (ἄρμόττον)

Contrast describing, telling explicitly, axiomatizing
and looking at how different variants of a structure
“fit” within one another, how they “reflect” in the
small properties of the large



CHANGE OF FOCUS: FROM FORMULAS TO EMBEDDINGS



φ, T, \dots	
formulas, theories	

CHANGE OF FOCUS: FROM FORMULAS TO EMBEDDINGS

φ, T, \dots	\prec_K
formulas, theories	embeddings, encasings ...

A WORLD OF PURE PHENOMENA...

... without precise descriptions, apparently, but with a strong notion of how pieces are fit within one another—ἀρμόττον

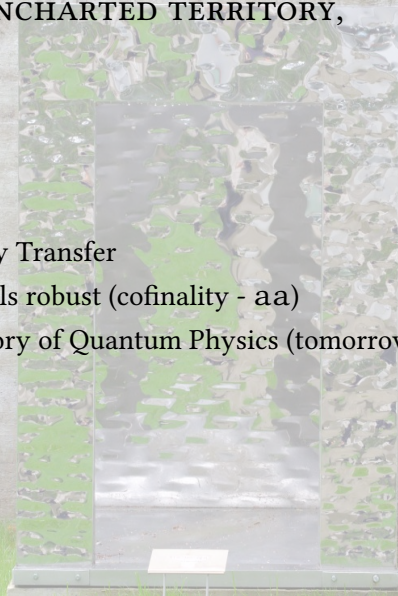
The name for that in contemporary model theory is “strong extension” $M \prec_K N$.

Roughly: all small configurations/problems from M that have a solution in N also have another solution in M .

SAILING INTO UNCHARTED TERRITORY,

... into the open.

- ▶ Categoricity Transfer
- ▶ Limit Models robust (cofinality - aa)
- ▶ Model Theory of Quantum Physics (tomorrow!)
- ▶ ...



THE PRESENTATION THEOREM

Yet even here, Logic seems to reappear!

Given any AEC $(\mathcal{K}, \prec_{\mathcal{K}})$ of structures in a language L ,

Also, $\prec_{\mathcal{K}}$ is controlled by the language!

THE PRESENTATION THEOREM

Yet even here, Logic seems to reappear!

Given any AEC $(\mathcal{K}, \prec_{\mathcal{K}})$ of structures in a language L ,

there exists a bigger language $L' \supset L$ in which you can write an

infinitary formula ψ that “holds” all the information on the class \mathcal{K} -

in our local dialect we say that \mathcal{K} is a PC-class for omitting types in

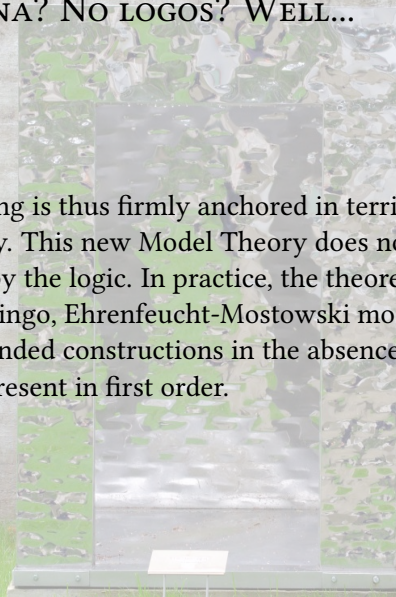
an expanded language,

$$\mathcal{K} = PC(L, T', \Gamma')$$

Also, $\prec_{\mathcal{K}}$ is controlled by the language!

PURE PHENOMENA? NO LOGOS? WELL...

The second sailing is thus firmly anchored in territory we may chart - albeit indirectly. This new Model Theory does not “reduce” to a class controlled by the logic. In practice, the theorem provides “steam” (in our lingo, Ehrenfeucht-Mostowski models, for example) to carry long-winded constructions in the absence of the tools (compactness) present in first order.



Prelude: Our Standing Point?

Syntax vs Semantics in Aesthetics?
○○○○○○○○○○

Meaning from language
○○○○○○○○○○

Language from meaning
○○○○○○○○●○



EXTERNAL / INTERNAL

So, back to Juliette's question: what is our standing ground as logicians? Why the incredible formalism freeness of mathematical practice?

EXTERNAL / INTERNAL

So, back to Juliette's question: what is our standing ground as logicians? Why the incredible formalism freeness of mathematical practice?

Thank you all!