Grasping smoothly and letting go: categoricity and location

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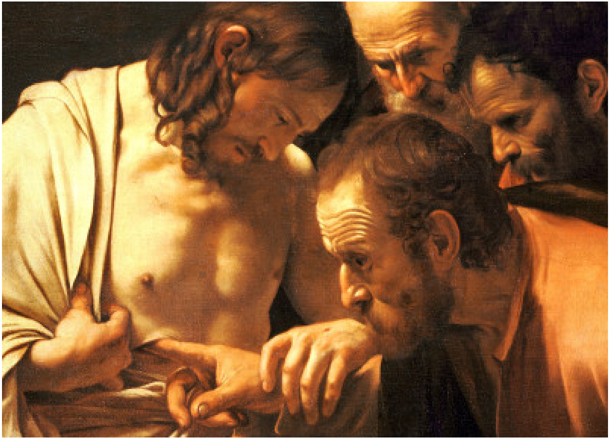


Framing disbelief

- The mystery of representation through language.
- Long, illustrious line: Plato, Aristotle, Ibn Rushd, Leibniz, Kant, Schopenhauer, etc.
- I will describe Categoricity, Analyticity/Holomorphism.
- Explore the "dynamics"
- Propose a role for Categoricity in the 21st Century

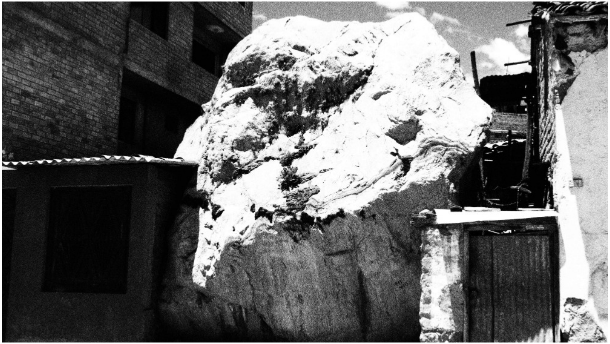
... YET FALLING SHORT

Categoricity (in power)

For L a first order language, T an L-theory and λ an infinite cardinal, T is categorical in λ if for every $M_1, M_2 \models T$, M_1 and M_2 are isomorphic.

Morley, Shelah, etc.

If a theory is categorical in some uncountable cardinality λ then it is also categorical at all uncountable cardinalities.

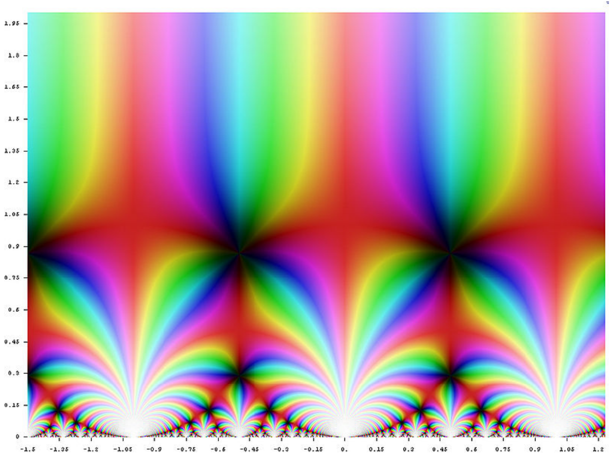


Novalis, the miner intellectual:

The affinity of geometry and mechanics with the loftiest problems of the human spirit, shines forth from the atomistic and dynamic sectarian strife. The painting of words and signs affords countless possibilities. One might envisage a perspective and manifold tabular projection of ideas, harboring the promise of infinite gain. Novalis, *Allgemeine Brouillon*

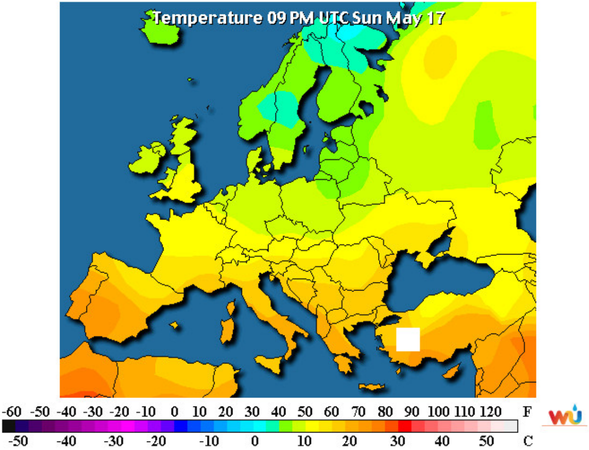
Analytic/Holomorphic functions

"Smooth", defined on a subset of the plane, complex-differentiable at every point of their domain.



If $f : U \to \mathbb{C}$ is holomorphic, then its values in the interior of any circle γ are determined by those in the boundary:

$$f(a) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(z)}{z - a} dz$$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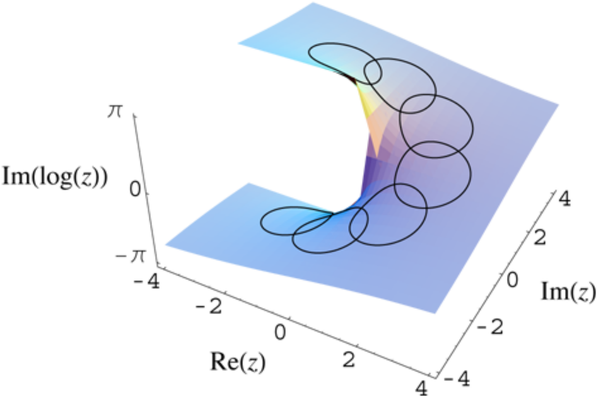
Identity Theorem

Given functions f and g holomorphic on a connected open set D, if $f = g$ on some open non-empty subset of D, then $f = g$ on D.

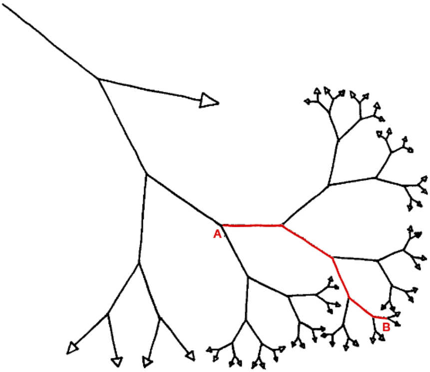
Analytic Continuation



(Graph by Yamashita Makoto)

Forking Paths - decision trees - categoricity proof

The Garden of Forking Paths
by Jorge Luis Borges



Topoi as places to study (even) categoricity

- A place, very abstract
- French school of Algebraic Geometry (Weil, Cartan, Leray, Grothendieck)
- Many phenomena beyond categoricity
- Quasiminimal Abstract Elementary Classes - pseudoexponentials, analytical functions, modular invariants (j mappings): the core of Analytic Number Theory.
- Music (Mazzola), Physics (Isham, Doering, etc.).

Obstructions, Poincaré, cohomology, sheaves, dynamics.

Thank you for your attention!