Temporality, Consciousness and Logic*

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*Enfre ·I· alquimista e el foch fo gran qüestió, car lo alquimista dix que artificialment pot hom simplificar los elaments, e depurar e depertir la ·I· element del altre, stant quescun element simple, per si mateix, cors simple, compost tan solament de una forma e de una matèria simple ab accidents simples. Molt se meravellà lo foch de la folla oppinió del alquimista (...) per açò es impossible que ·I· element pusque ésser sens altre; car si ·I· element podia esser sens l'altre, porie ésser l'aer humit per si mateix ...** Libre de meravelles, vol. II, XXXVI - Ramon Llull

1. Phenomenology, reality, invariance

A crucial feature of phenomenology** may be described as the study of "*that* which remains *invariant* under changes of *reality mode*". This may be different from standard descriptions, but it captures a crucial point. When we transition

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**There was a great dispute between an alchemist and Fire. Indeed, the alchemist claimed that one could artificially simplify the elements, and depurate and separate one element from another, each element being simple in itself, a simple body, composed only of a form and of a simple matter with simple accidents. Fire was greatly astonished by the mad opinion of the alchemist (…) for it is impossible for an element to be without another; for if one element could be without another, air could be humid in and of itself…* The Book of Marvels, vol. II, XXXVI - Ramon Llull.

**I owe an important part of this perspective to private conversations with Mark Ettinger.

from being awake to an oneiric state or to some simulation of reality or to some illusion, we may say we are switching **reality mode**. While some experiences may not be invariant under change of reality mode, others will be. For instance, if we perceive in our dreams (or the semi-vigil of some mathematical proof at the moment of its conception, or in a state of ebriety) that some book *is blue,* the experience of perception of being blue and in particular its rooting in spatialness turns out to be invariant under changes of reality mode: it simply is not possible to conceive the color blue without *some form* of spatiality. This dependence on spatiality is invariant under reality mode.

Now, phenomenology allows us to take this example further: it doesn't just grasp these invariant entities (under change of mode of reality) but it purports to see *how* they are anchored in some other category. For example, as much as we may attempt (awake, in a dream, in ebriety, in mathematics) it, we may not really disassemble "blue" from some kind of spatiality. There simply is no blue that is not extended in space. The notion *being blue,* whether we consider (awake) the blue of some book or (in dreams) that of a unicorn, seems to be *hooked* in an unavoidable way to the concept of spatiality. Even if the blue spot we see is tiny, it still occupies some kind of space.

This *dependence* of blue with respect to spatiality is an example, maybe very immediate and primitive, of what Husserl called Fundierung ("being based on, or funded upon…"). Fundierung is one of the most important and primordial points of what we could call fundamental phenomenology.

We may point to many other examples of dependence relations essential to our consciousness that in some way or another constitute this version of Husserl's Fundierung. Rota points to [3], for example, the relationship between the *function* of the queen of hearts (in poker, in bridge) and the physical card. This is a type of Fundierung quite differeng from that of the color blue with respect to spatiality. The other example (originally, Wittgenstein[7], reworked by Rota) is reading. The movements of the eyes, the neurons activated while we read, the spelling of words letter by letter, the understanding of each individual word, etc. clearly are not **sufficient** to "univoquely determine that a person *is reading*." The *function* of reading in this case has a Fundierung relation with the *process* of reading (the event that happens in space and time, built from microevents that succeed one another in time).

This is the *main question* arising since the time of Husserl, as reformulated by Rota [7] in slightly different terminology:

Which logic governs the Fundiering relation?

Let us now start threading in a finer way the question, by using an analogy-with the revolution that Propositional Calculus entailed more than two and a half millennia ago* The point is simple: at some moment it became evident that the essential point of phrases such as "if all A is B and all B is C then all A is C" does not depend on the contents of A or B or C but rather always holds for structural reasons. Although today this fact is internalized since tender ages**, it embodied an authentic *revolution* when it happened. The rest of the story is well known and has been amply studied. Its main offspring is contemporary mathematical logic.

However, a first very radical step in the sense of *not taking the "world" for granted*, as something obvious, was needed in order to extract, two and half millennia ago, the *structural* character of statements like the one above. In an analogous (but infinitely more complex) way, Husserl has offered a problem to mathematical logicians *and to philosophers:* constructing, defining, building a mathematical version of transcendental logic.

The response to this challenge has been very tenuous and shy until now, especially from the perspective of mathematics. Some authors [4] have posited Tarski's original version of model theory as the mathematical rsponse to Husserl's transcendental logic. Although there are reasons to see the satisfaction relation $\models$ as a beginning of a response, this is clearly only a very partial response, very much just *on the way* to a true transcendental logic or a logic of Fundierung.

In what follows I will argue why sheaf logic, the logic of topoi and finally the "architectural construct" called THK by Zalamea[9] is a very solid beginning of a mathematical response to the problem of pinning down what really happens with transcendental logic. I also show in which sense are we still far from a more complete response to Husserl's question.

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*The analogy, which seems so "obvious" and basic to us mathematical logicians, has been used many times by many authors (among them, very much *avant la lettre*, Charles Saunders Peirce, as Zalamea and Oostra have made clear in many of their works). In this case it is useful.

**Or at least, so we would like to believe. A quick browse of dominant contemporary journalistic material might convince us that not even Propositional Calculus has permeated global culture, after 2500 years.

2. Awareness of Internal Time - Transcendental Logic

One of the origins of the quest that ends up in Transcendental Logic in Husserl is the philosophical analysis of consciousness of internal time, the access and perception we may (or not) have of time's unfolding. If we accept as a main role for logic to provide an objective anchoring to our perceptions (mediated, modalized, filtered) and be a place of synthesis of all our ("noematic") acts of knowledge, it is clear that a logical relation adapted to Fundierung necessarily has to account (among other things) for the way we access the effect of time, the way we perceive it.

Husserl's analysis of time perception is immensely rich in examples, ideas and layerings of these, contrasting *memory of past events* with *intentionality* (the way our thinking directs itself toward an event and effectively manages to *interpolate* between visible instances of those events). For instance, if I see my hand rotating I may only perceive *some of the states* of this rotation (probably very few of them) but through intentionality I may "interpolate" to a continuum of intermediate stages and see ONE rotating hand - I do not interpret it as a distinct object in each one of the moments I perceive through sight. This way of "glueing" or synthetizing* is a crucial element of the analysis of time. Another crucial element, connected with (but not reducible to) the previous is *protention*, the "anticipation" or bounded extrapolation of the movement or temporal event. (This point has been much less analyzed mathematically; I expand on this point later.) An immediate version of this protention is that in some sense I may extend (my perception of) time beyond, "I overflow" the perceived instant and anticipate that the hand will "still be there" at least in a very close future. I *anticipate,* although in a very slight way. From these two elements, Husserl builds a network of time decomposition in a *continuum* of anticipations and interpolations, protentions and intentionalities, extremely refined and irreductible to a merely analytic description.

The role of logic is ultimately to provide a "place" for a common anchoring of all possible experiences of the world. Husserl constructs basically two levels of logic: formal and transcendental. He further analyzes formal logic in two sublevels. Sokolowski explains ** these three logics as the Logic of Consistency, Logic of Truth and finally Logic of Transcendence. There are very strong Peir-

*central for Peirce as well

**Private communication with the author and Mark Ettinger.

cean echoes (risking here extreme reduction) in this analysis of Husserl's logic: *consistency logic* expresses our direct, unmediated, immediate access to perceived things; *truth logic* somehow addresses the direct and critical judgment we may make through the use of language, and *transcendent logic* (the least well-defined, the least studied of the three logics) involves processes of **reflection** and contrast between the two previous levels.

There is really much more in transcendent logic than this description in Peircean terms; it involves temporality, it involves a stepping beyond language and direct objectuality (hence the name "transcendental" of that logic). The scope of these notes does not cover the philosophical complexities Husserl posits - almost in the style of a mathematical problem for the coming decades, reformulated by Rota.

However, we may since now state a few important points with respect to the problem, and leave for future work the completion of this description.

3. THK: A first transcendent logical calculus?

In recent writings (and, I believe, in some other essays to appear soon), Zalamea puts forth his **THK** theory, called this way because of the emblematic initials (in Spanish) of the words Topos, Sheaves (Haces) and Kripke. This theory incorporates the following three levels (or interconnected surfaces):

- A "base" level **K** (Kripke): a beginning of "temporality" (still at a much more primitive level than in Husserl, but already much more complex than Tarskian semantics),
- An "intermediate" level **H** (sHeaves): coherence and glueing - a beginning of the synthetic substratum in Husserl (intentionality)
- An "upper" level **T** (Topos): the emergence of topos logic - the point where transcendent logic may be started (stage I).

Husserl has used the term *manifold* or multiplicity to refer to the semantic aspect of his own formal logic, and to initiate the path to transcendental logic. Sokolowski proposes* to identify the concepto of "model" in the Tarskian sense and Husserl's "manifold". I believe this is an interpretation that goes in the correct direction but is quite limited.

*in private correspondence with the author and with Mark Ettinger

As a matter of fact, the Tarskian notion of model is indeed "manifold" when filtered through Gödel's Completeness Theorem. The multiplicity of models of a theory, a first variant of the manifold aspect of semantics, appears naturally. In this precise sense Sokolowski is right. I see however limitations in the following points:

- Nowhere in Tarskian models is there the possibility of incorporating anything akin to time perception in the extremely refined sense of Husserl's.
- Whenever compactness is present in a system there are sheaf-like properties (global/local principles, the hallmark of compactness, albeit usual compactness does not reveal the eminently geometric character of those interactions); however, most logical systems lack compactness.
- Theories of symbiosis (instantiated in many different ways, but inspired in primordial symbiosis between logics of arbitrarily high orders and set theory) due to Bagaria and Väänänen [5] and the notion (due to Väänänen [6]) of Internal Categoricity have to be included in any attempt at transcendence in the Husserlian sense. These two notions allow transitions between many different types of logics with the immense flexibility of set theory and allow to carry the powerful tools of the latter much beyond their usual ambits.
- The myriad ideas of Grothendieck's need to be incorporated as well (not just topoi; also the tools that have enabled an understanding in an "archetypical integral" way —to paraphrase Zalamea— obstructions to categoricity given by the "generalized cohomologies" in **motives**) in the construction of transcendental logics.
- Finally, it will be crucial to adopt a very flexible perspective of "back-and-forth" between Zalamea and Shelah, between motives and proper forcing, between geometry and contemporary model theory construed as an "ultrageometry".

It is in this very way that Zalamea'a THK theory may be regarded as initiating the path toward a mathematization of Husserl's transcendental logic. Husserl's time continuum is rather more complex (in its articulation between protention and intention) than what Kripke models, or even sheaves and topoi, may yield. But if there is a path toward an actual mathematization of Husserl's transcendental logic, it will have to incorporate those notions (as well as symbiosis [5] and internal categoricity [6]).

4. A conclusion (pessimistic *ma non troppo*)

The richness of Husserl's questions on the transcendental anchoring of the world has not really been so far matched by the mathematizations of logic. There is so far no genuine transcendental mathematical logic; Rota's question has not truly been answered until now. This is a pessimistic aspect of the conclusion.

On the other hand, the (other kind of) richness of Grothendieck's and Shelah's mathematical constructions and their parallel schools (and their future entanglement) had so far almost not been picked up or even recognized by philosophers as an object of study... certainly not in a radical way until the appearance of works such as [8]. This book stands rather alone among philosophical studies of the subject; there are not many more analyses of such constructions at their most sophisticated levels, at least not in a global synthetic way. Some mathematicians have started this path relatively recently ([1], [2], [10] are examples). However, up to now even the work of Baldwin, Harris or Zilber, those three mathematical towering figures, has been much more localized in its application to philosophy than Zalamea's work.

But going back to the point of pessimism, here is the reason for the *ma non troppo* twist I use to modulate it: in Zalamea's work on THK there is already the beginning of a path that promises to be very long, to provide at some point an answer to Rota's question –providing a logic for Fundierung.

I conclude these notes going back to the invocation of the great master of logicians, Ramon Llull. Quoting his sentence again, *es impossible que ·I· element pusque ésser sens altre, car si ·I· element podia esser sens l'altre, porie ésser l'aer humit per si mateix...*, it is impossible for one element to be without another, for if one element could be without the other, air could be humid *in and of itself*. It is as absurd to conceive humid air "in and of itself", with no Fundierung relation, as conceiving a non-spatial blue would be. Llull is, perhaps poetically, alluding to a notion we could read today in Fundierung mode.

Llull was almost alone in this path as he opened (he was however standing on the shoulders of Abulafia and other Talmud scholars) his system to use logic as a tool to understand problems of the world and our connection to it, of possibilities and impossibilities. Our Llulian logician Fernando Zalamea has undertaken a kind of parallel path, very lonely as well, and he has also raised questions that his contemporaries do not fathom –just as Llull. Those of us who have had the immense fortune of crossing our shaky paths with Zalamea's deep wake are a but similar to those travelers Llull must have met when he sailed across his Mediterranean Sea to bring his ideas and books. Lucky fellow travelrs in the giant

and dangerous sea of difficult times.

Referencias

[1] Baldwin, John, *Model Theory and the Philosophy of Mathematical Practice: Formalization without Foundationalism*. Cambridge University Press, 2018.

[2] Harris, Michael, *Mathematics without Apologies: Portrait of a Problematic Vocation (Science Essentials)*. Princeton University Press, 2015.

[3] Rota, GianCarlo, *Fundierung as a logical concept*, en **Indiscrete Thoughts**, ed. F. Palombi, Birkhäuser, 2009.

[4] Smith, David Woodruff, *Husserl (The Routledge Philosophers)*. Routledge, 2013 (2nd edition).

[5] Bagaria, Joan y Väänänen, Jouko, *On the symbiosis between model-theoretic and set-theoretic properties of large cardinals*. Journal of Symbolic Logic 81(2), 584-604 (2016).

[6] Väänänen, Jouko, *Tracing Internal Categoricity*, to appear (2019).

[7] Wittgenstein, Ludwig, *Investigaciones Filosóficas*. Trad. Jesús Padilla Gálvez. Ed. Trotta, 2017.

[8] Zalamea, Fernando, *Filosofía Sintética de las Matemáticas Contemporáneas*. Bogotá: Editorial Universidad Nacional de Colombia, Colección Obra Selecta, 2009.

[9] Zalamea, Fernando, *Discurso de Recepción como Académico Honorario*, Academia Colombiana de Ciencias Exactas, Físicas y Naturales, 2018.

[10] Cruz, Alexander; Villaveces, Andrés y Zilber, Boris, *Logical perfection in mathematics and beyond*, to appear (2019).