Conjeturas estándar y teoría de modelos

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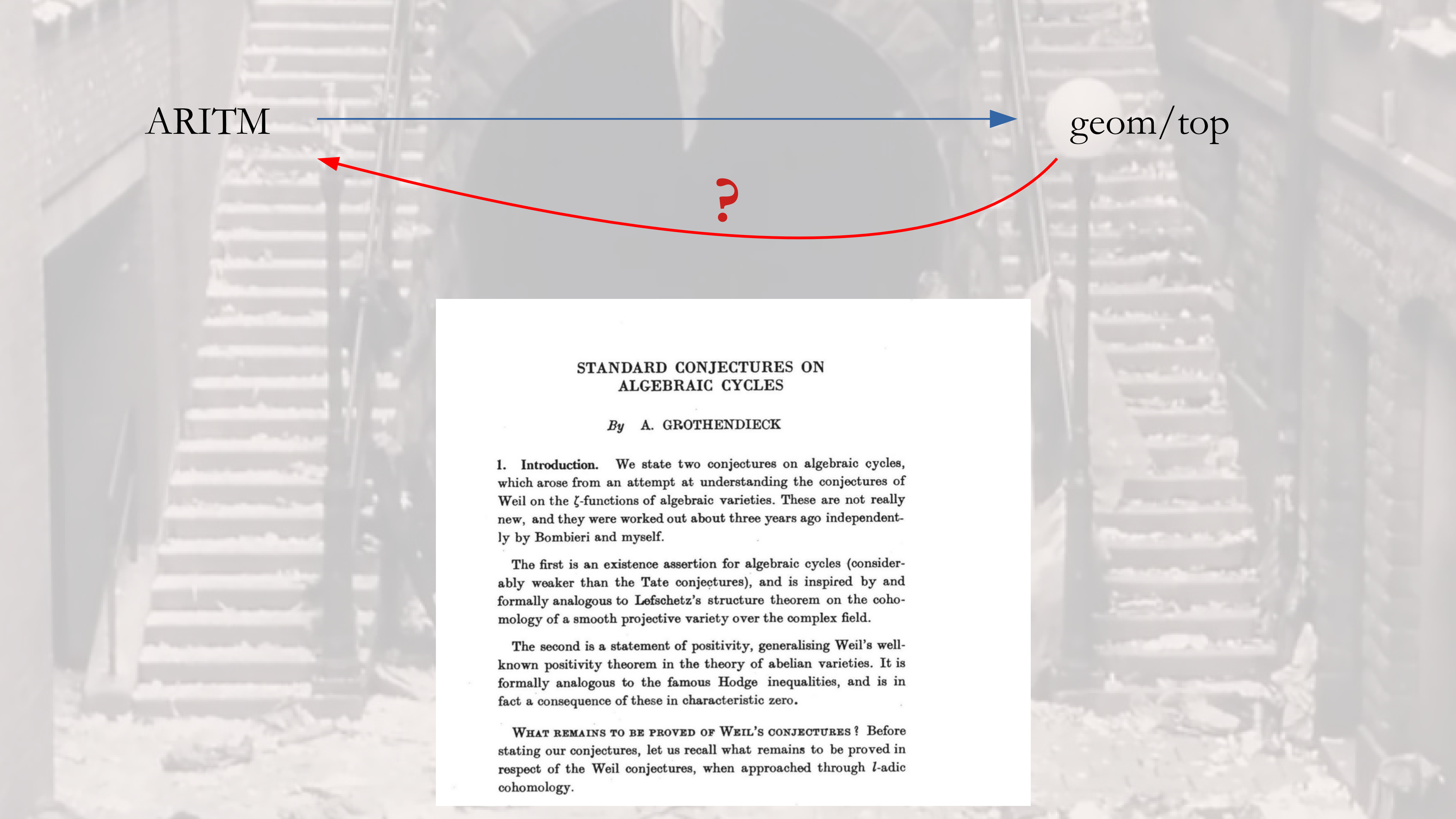
Tertulias Matemáticas: Resolución de las Conjeturas de Weil (Grothendieck/Deligne, 1958-1974)

Dos imágenes

(El sacrificio, Tarkovski. Fotografías: Sven Nykvist)



STANDARD CONJECTURES ON
ALGEBRAIC CYCLES

By A. GROTHENDIECK

1. Introduction. We state two conjectures on algebraic cycles, which arose from an attempt at understanding the conjectures of Weil on the ζ-functions of algebraic varieties. These are not really new, and they were worked out about three years ago independently by Bombieri and myself.

The first is an existence assertion for algebraic cycles (considerably weaker than the Tate conjectures), and is inspired by and formally analogous to Lefschetz's structure theorem on the cohomology of a smooth projective variety over the complex field.

The second is a statement of positivity, generalising Weil's well-known positivity theorem in the theory of abelian varieties. It is formally analogous to the famous Hodge inequalities, and is in fact a consequence of these in characteristic zero.

What remains to be proved of Weil's conjectures? Before stating our conjectures, let us recall what remains to be proved in respect of the Weil conjectures, when approached through l-adic cohomology.

Una descripción **modelo-teórica** de las conjeturas ha ido surgiendo

Matemática conjetural → Teoría de las conjeturas

Misha Gavrilovich

Las conjeturas estándar como la búsqueda de una
definición *puramente algebraica* de:

- cohomología singular (de Betti) o
- el grupoide fundamental topológico

(de una variedad algebraica compleja)

Standard Conjectures in Model Theory, and Categoricity of Comparison Isomorphisms. A Model Theory Perspective.

Superposición de conjeturas, para entender mejor las **conjeturas originales**

La lectura modelo teórica iniciada por Zilber y generalizada por Gavrilovich:
Sea un funtor (e.g. $\pi_1^{top} : \mathcal{V}ar \to \mathcal{G}roupoids$);
queremos saber cuándo tiene definición puramente algebraica:
Si π_1^{top} y $\pi_1^{top} \circ \sigma$ coinciden con una tal definición,
para todo $\sigma : \mathbb{C} \to \mathbb{C}$ automorfismo de campos.

La clave modelo-teórica es encontrar una definición del funtor π_1^{top} **posiblemente enriqueciendo el lenguaje o usando distintas teorías de modelos** de manera tal que la descripción algebraica capture de manera unívoca el funtor, módulo automorfismos de $\mathbb{C}$.

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$$(*) \qquad \cup\, \xi^{n-i} : H^i(X) \longrightarrow H^{2n-i}(X) \quad (i < n).$$

It is expected (and has been established by Lefschetz [2], [5] over the complex field by transcendental methods) that this is an isomorphism for all characteristics. For $i = 2j$, we have the commutative square

$$\begin{array}{ccc} H^{2j}(X) & \xrightarrow{\xi^{n-2j}} & H^{2n-2j}(X) \\ \uparrow & & \uparrow \\ C^j(X) & \longrightarrow & C^{n-j}(X) \end{array}$$

Our conjecture is then: $(A(X))$: (a) (*) *is always an isomorphism (the mild form);*
(b) *if $i = 2j$, (*) induces an isomorphism (or equivalently, an epimorphism)* $C^j(X) \to C^{n-j}(X)$.

N.B. If $C^j(X)$ is assumed to be finite dimensional, (b) is equivalent to the assertion that $\dim C^{n-j}(X) < \dim C^j(X)$ (which in particular implies the equality of these dimensions in view of (a).

An equivalent formulation of the above conjecture (for all varieties X as above) is the following.

$(B(X))$: *The Λ-operation (c.f. [5]) of Hodge theory is algebraic.*

Horosis modelo-teórica para la geometría y la aritmética...

rational coefficients *independent of l*, holds.

Conclusions. The proof of the two standard conjectures would yield results going considerably further than Weil's conjectures. They would form the basis of the so-called "theory of motives" which is a systematic theory of "arithmetic properties" of algebraic varieties, as embodied in their groups of classes of cycles for numerical equivalence. We have at present only a very small part of this theory in dimension one, as contained in the theory of abelian varieties.

Alongside the problem of resolution of singularities, the proof of the standard conjectures seems to me to be the most urgent task in algebraic geometry.

Ascenso modelo-teórico (**categoricidad** *ideal*) para CALIBRAR los funtores:

- ¿Buscar que $F_{top} = F_{top} \circ \sigma$ para todo $\sigma \in Aut(\mathbb{C})$? CASI
- Expandir el lenguaje (y en algunos casos pasar a lógica infinitaria $L_{\omega_1,\omega}$ o **clases elementales abstractas**) para
- capturar teorías consistentes de equivalencia de funtores…
- Posiblemente en modelos grandes (el "campo de Zilber" en lugar de la exponencial compleja, el módulo de Tate en versiones asociadas a funciones modulares)
- Leyendo modelo-teóricamente la ecuación funcional que gobierna la conjetura, y
- Usando técnicas modelo-teóricas (excelencia, cuasiminimalidad) nacidas en la obra de Shelah, para capturar la categoricidad

Ejemplos: pseudoexponencial compleja de Zilber, cohomología singular (Bays, Gavrilovich, Kirby, Zilber), teorema de la Aplicación Abierta de Serre (categoricidad en $L_{\omega_1,\omega}$), etc.

¡Gracias!

Continuamos la **tertulia...**