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Model Theory of non-elementary classes: the role of covers

Montevideo
XX SLALM

*: state
of
the
art

- Brief overview of Stability Theory for AECs — up to stable / simple. *
- Landmarks of this development, the NIP area.
- Most important examples so far:

- covers
- why AEC?
- Some of what there is

- modules *
- covers *

① Covers

HT	Geom.	A kind of description (Zilber, S. Con. GALois)
For categoricity, stability	Alg. Geom.	
?	Analytic (complex) geom.	
AEC categ. / stab.	?	
$L_{w,w}(\cdot)$ categ.	Analytic theory	

Prototypical example: $\mathbb{C}_{exp} = (\mathbb{C}, +, \cdot, exp)$

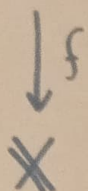
2]

This required $L_{\text{new}}(Q)$, involved ^⑥ transc. number theory (eg period conjecture)

⑥
check

More relevant / more basic: COVER structures.

①



(for complex algebraic geometry,

cover structures)

+ alg. subvar. of X^n ($\forall n$), definable.

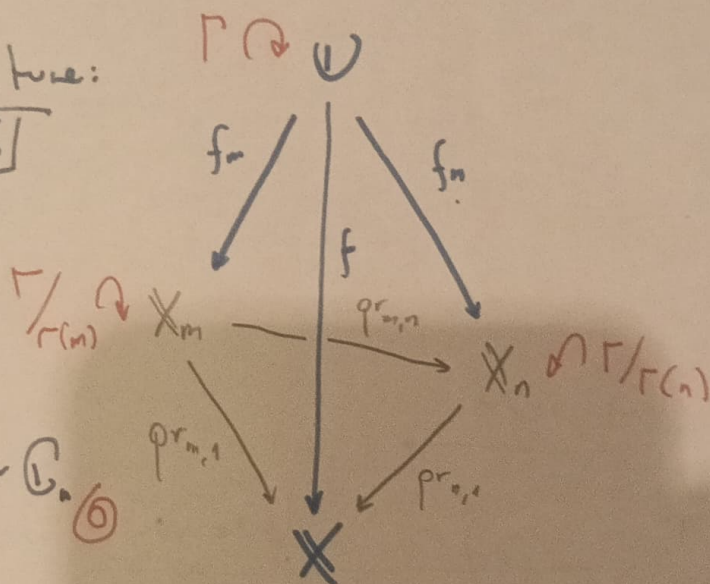
The cover structure:
[CS]

For example,

$$U = \mathbb{C}$$

$$f_n(z) = \exp(z/n)$$

$$X_n = \mathbb{C}^* \quad \Gamma/\Gamma(n) \cong \mathbb{C}_n$$



Point: the CS holds info on the metric topology of $X(\mathbb{C})$
 $\searrow$ stable [alg. geom.]

W/o the cover, only have the étale topology on $X \rightarrow X$ up to abstract autom. of $\mathbb{C}$.

Remark: Existence of non-trivial covers is a strong requirement on X . Other?

Large Theorem (Zilber, Kirby, Gavrilovich, Bays, Harris, Daw, Hart, Haykazian, Hyttinen, Eterović...) [3]

The natural theory (in $L_{w,w}$) of covers

for $\left\{ \begin{array}{l} \exp_{Th_{w,w}(p)} \\ j \\ \text{modular/Shimura curves} \\ \text{--- -- var.} \\ \text{Abelian varieties} \\ \text{finite groups} \\ \text{smooth varieties} \end{array} \right\}$ is categorical in uncountable cardinals.

Proofs (2005, 06, 11, 17, 20, 22)

require
plus

model th. of AECs
strong arithmetic theorems
e.g. Mumford-Tate Conj. (proved versions)
Galois/Kummer theory (extended)

[The theorem is being extended — see our monograph with Baldwin — by Harris (H.),
Cruz, V., Baldwin, Zilber mirror maps

Moreover, if $Th_{w,w}(p)$ (natural) is ^{uncountably} categorical, the arithmetic level facts follow!

4]

Cat. of
 $\text{Th}_{w,w}(j)$

M.Th.

$\longleftrightarrow$
M.Th. GAGAGA

An. thm.

Facts

(Mumford-Tate...)

A. G

Complete Topological invariants

$$\sum (X_a)$$

$$\sum (X_a) = \sum (X_a)^\sigma$$

$$\downarrow$$

$$X_a^\sigma \approx X_a \text{ or } X_a^{\text{cc}}$$

for complex (Shimura, Abelian) varieties

$$X_a$$

Already: for all smooth complex proj. curves
 X of genus ≥ 1 , there is an $L_{w,w}^{(Q)} \text{-ax.}$

$$\sum (X)$$

categorical in all unctble cardinals.

Ingredients: o-minimal structure on the w -var

+ Boggs - Hart - Hyttinen - Kestölä - Kirby

2. AECs

- Early days: "oly. minded model th." (Shelah 1974)
- Analy. Clones (Fraïssé, Jonsson)

Baldwin, to Libgober:

for what's worth, AEC

LS

one a style of model theory that approaches mathematics in a more function fashion. Instead of syntax & semantics, one investigates class of structures that satisfy certain properties (like closure under limits) — ^{indeed} there is a pure category theory def'n.

AEC: smoothly forward closed

generative

coherent (e.c.)

model classes

$$M_i \preceq_{\mathcal{K}} \text{-class} \Rightarrow \bigcup M_i \in \mathcal{K}$$

$$M_i \preceq \bigcup M_i$$

$$\bigcup M_i = \sup M_i$$

Löwenheim-Skolem-Tarski # exists

$$\begin{matrix} \tilde{M} \\ \nearrow \searrow \\ M \subseteq N \end{matrix} \Rightarrow M \preceq_x N$$

$$\begin{matrix} \mathcal{K} \\ \downarrow \\ \tau\text{-structure} \end{matrix} \quad , \quad \begin{matrix} \preceq_x \\ \downarrow \\ \text{order of } \mathcal{K} \end{matrix}$$

$$\begin{matrix} M' \preceq M \in \mathcal{K} \\ \Downarrow \\ M' \in \mathcal{K} \end{matrix} \quad \begin{matrix} N' \approx N \\ \cup \\ M' \approx M \end{matrix} \Rightarrow \begin{matrix} N \\ \tau'' \\ M' \end{matrix}$$

$\mathcal{F}_0$, compl.

xxx: $(\text{Mod}(\tau), \preceq)$

$(\text{Mod}(\varphi), \preceq)$

L_{KW} $(\text{Mod}(\varphi), \preceq_{\mathcal{F}})$

$(\text{loc. fin. groups}, \preceq_{\mathcal{F}})$

$(\text{Noetherian rings}, \preceq_{\mathcal{F}})$

(Part I examples)

$(\mathcal{R}\text{-Mod}, \preceq_p)$

$\vdots$

Although no (directly) def'ble set,
strong grip on typespaces $gS(M)$.
(Galois types)

In some AECs (AP, NMM, JEP)

$$\text{gtp}(a/M, N) = \text{gtp}(b/M, N)$$

$$\Leftrightarrow \exists f \in \text{Aut}(M) \quad \begin{matrix} f(a) = b \\ f \upharpoonright M = \text{id}_M \end{matrix}$$

* Stability now pinned down.
(true) (2017)

(uniqueness of "limit models")

stability in large spectra

V.
VD
Grossberg
...

* stable forking $M_A \perp^N$

has a Kim-Pillay K charact.

(Grossberg ... 2016)

* Simple AECs started

* NIP AECs started

Mention Generic Pairs!

[u def.]